2023

माध्यमिक शिक्षा मण्डल, मध्यप्रदेश, भोपाल

32 पृष्ठीय

परीक्षार्थी द्वारा भरा जावे ↓

परीक्षा का विषय	विषय कोड	परीक्षा का माध्यम
Mathematics	8 5 0	English

स्टीकर तीर के निशान ↓ से मिलाकर लगायें

माध्यमिक शिक्षा मण्डल, म.प्र. भोपाल

MADHYAPRADESH

C-23

परीक्षार्थी का रोल नम्बर

अंकों में

2	3	3	1	3	9	2	0	5	-

शब्दों में

Two	Three	Three	One	Three	Nine	Two	Zero	Five	-

उदाहरण अनुसार रोल नम्बर भरें।

प्रश्न पत्र का सेट D

क :- परीक्षार्थी का कक्ष क्रमांक H911

ख :- परीक्षा का दिनांक 21 | 03 | 2023

परीक्षा केन्द्र नाम एवं परीक्षा केन्द्र क्रमांक की मुद्रा

SGH Code 311219

पर्यवेक्षक का नाम एवं हस्ताक्षर	केन्द्राध्यक्ष / सहायक केन्द्राध्यक्ष के हस्ताक्षर
Nitin Deshmukh	

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे ↓

प्रमाणित किया जाता है कि होलो क्राफ्ट स्टीकर क्षतिग्रस्त नहीं पाया गया तथा अन्दर के पृष्ठों के अनुरूप मुख्य पृष्ठ पर अंकों की प्रविष्टी एवं अंकों का योग सही है।

निर्धारित मुद्रा : नाम, पदनाम, मोबाईल नम्बर, परीक्षक क्रमांक एवं पदांकित संस्था के नाम की मुद्रा लगाएं।

उप मुख्य परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा	परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा
V.No. 4753	V.N.-
	L.P. Mahore V.No.-015305

केवल परीक्षक द्वारा भरा जावे।

प्रश्न क्रमांक के सम्मुख प्राप्तांकों की प्रविष्टी करें।

प्रश्न क्रमांक	पृष्ठ क्र.	प्राप्तांक
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प्र क.

Ans of Q.1

s $125|A|$

Ans ~~not defined~~ not defined

± 6

Ans $\cancel{\pi/3}$ f is one-one onto

Ans $-\pi/3$

ii Ans $\pi/3$

B

Ans of Q.2

i Ans $\frac{\pi \times x}{2}$

ii $\frac{\pi}{3}$

zero

$(1,0,0)$, $(0,1,0)$ and $(0,0,1)$

$e^{x} f(x) + C$

$$\frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}} = \frac{\sqrt{(e^{\sqrt{x}})}}{4\sqrt{x}}$$

$10\pi\ cm^2$ | cm

Ans of Q.3

Derivative of $\sin 2x$ with respect to x — $2\cos 2x$

3

योग

(2) $\int \tan x \, dx$ — $-\log|\cos x| + c$

(3) $\int \cot x \, dx$ — $\log|\sin x| + c$

$\int \sec x \, dx$ — $\log|\sec x + \tan x| + c$

$\int \operatorname{cosec} x \, dx$ — $\log|\operatorname{cosec} x - \cot x| + c$

$\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix}$ — $\cos 2x$

Ans of Q.4

i Ans 1

~~0.12~~ 0.4

0

126

$\pi/4$

ii)

iii) A matrix containing single column is known as column matrix like $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Ans of Q.5

i Ans True

ii Ans True

iii Ans False

प्रश्न 5.

Ans False

Ans True

Ans False

Sol: Q.6

Given,
direction ratios of line are $-18, 12, -4$

Let these direction ratio are a, b, c respectively

Then
direction cosines will be

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \quad \frac{b}{\sqrt{a^2+b^2+c^2}} \quad \text{and} \quad \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Here $a = -18 \quad b = 12 \quad c = -4$

So

$$\sqrt{a^2+b^2+c^2} = \sqrt{(-18)^2+(12)^2+(-4)^2}$$

$$= \sqrt{324+144+16}$$

$$= \sqrt{484}$$

$$= 22$$

So
direction ratios are

$$\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} \quad \text{i.e} \quad \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$

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(i)

29   02   31

Sol Q.7   OR

Given

$f(x) = x^2 - 4x + 6$

So $f'(x) = 2x - 4$

$= 2(x-2)$

For decreasing function $f'(x)$ should be < 0

So $f'(x) < 0$

$2(x-2) < 0$

$\Rightarrow x - 2 < 0$

$\Rightarrow x < 2$

So the given function will be decreasing in the interval

$(-\infty, 2)$

Sol: Q.9   OR

To prove - $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ $x \in [-1, 1]$

Let $\sin^{-1} x$ ~~be~~ $= A$ — eqn (1)

$\Rightarrow x = \sin A$

$\Rightarrow x = \cos\left(\frac{\pi}{2} - A\right)$ $\left\{\cos(90-\theta) = \sin\theta\right\}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - A$ — eqn (2)

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On adding eqn ① and ②
we get

$\sin^{-1} x + \cos^{-1} x = A + \frac{\pi}{2} - A$

$\Rightarrow \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

Hence proved

Solution Q.10

Given

$A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

So
$A + B = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -3 & 3 \\ 2 & 4 \end{bmatrix}$

now,
$(A+B)' = \begin{bmatrix} -3 & 2 \\ 3 & 4 \end{bmatrix}$ Ans

(7)

प्रश्न क्र.

Sol: Q 11 OR

Givn

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

To find $\frac{dy}{dx}$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$ — eqn-①

So

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$ $\left\{\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right\}$

$\Rightarrow y = 2\theta$

$= 2\times\tan^{-1}x$ { from eqn-① }

Now

differentiating w.r.t x, we get

$\frac{dy}{dx} = 2\frac{d}{dx}\tan^{-1}x$

$\frac{dy}{dx} = 2\times\frac{1}{1+x^2}$

$\frac{dy}{dx} = \frac{2}{1+x^2}$ Ans

B
S
E

8

प्रश्न :

Sol: Q.12

Given
$f(x) = \cancel{12x}\ 12x - 3$

So $f'(x) = 12$

We know that
any function $f(x)$ is increasing if $f'(x) > 0$

Here $f'x = 12$ i.e > 0
Hence $f(x)$ is an increasing function in the interval of R

Sol. Q.13 OR

To find $\int_{-1}^{2} |x^3 - x|\, dx$

We know that
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

So
$$\int_{-1}^{2} |x^3 - x|\, dx = \int_{-1}^{2} x^3 dx - \int_{-1}^{2} x\, dx$$

प्रश्न क्र.

$$= \left[\frac{x^4}{4}\right]_{-1}^{2} - \left[\frac{x^2}{2}\right]_{-1}^{2}$$

$$= \frac{2^4}{4} - \frac{(-1)^4}{4} - \left\{\frac{2^2}{2} - \frac{(-1)^2}{2}\right\}$$

$$= \frac{16}{4} - \frac{1}{4} - \frac{4}{2} + \frac{1}{2}$$

$$= \frac{16-1-8+2}{4} = \frac{18-9}{4} = \frac{9}{4}$$

Sol Q.14.

Given,

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ = $a\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Now

$\vec{a}\cdot\vec{b} = (\hat{i}+2\hat{j}+3\hat{k})\cdot(2\hat{i}+0\hat{j}+\hat{k})$

$= 2+0+3 = 5$

$|\vec{b}| = \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$

$a\cos\theta = \frac{\cancel{2\hat{i}+\hat{k}}}{\sqrt{5}} \quad \frac{5}{\sqrt{5}} = \sqrt{5}$ Ans

प्र॰ क्र.

~~→ Projection of vector $\vec{a}$ on $\vec{b}$ will be~~

$$\cancel{\frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{k}}$$

Sol Q.15.

Given
A vector $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector

Let

$$x(\hat{i}+\hat{j}+\hat{k}) = \vec{r}$$

$$\Rightarrow \vec{r} = x\hat{i} + x\hat{j} + x\hat{k}$$

Now $|\vec{r}| = \sqrt{x^2+x^2+x^2}$

$$= \sqrt{3x^2}$$

$$|\vec{r}| = x\sqrt{3} \quad \text{— eqn — ①}$$

Now

~~we kno~~

According to question
$\vec{r}$ is a unit vector
So $|\vec{r}| = 1$ — eqn — ②

comparing eqn ① and ②
we have

$$x\sqrt{3} = 1$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}$$

The value of x is $\frac{1}{\sqrt{3}}$

Sol. Q. 16 OR

Given differential eqn. is

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

$$\Rightarrow \frac{1}{1+y^2}\, dy = \frac{1}{1+x^2}\, dx$$

On integrating we get

$$\int \frac{1}{1+y^2}\, dy = \int \frac{1}{1+x^2}\, dx$$

$$\Rightarrow \tan^{-1} y = \tan^{-1} x + C \quad \left\{ \int \frac{1}{1+x^2}\, dx = \tan^{-1} x \right\}$$

So the required general solution is

$$\tan^{-1} y = \tan^{-1} x + C$$

Sol Q.18

Given

Urn contains 10 black and 5 white balls

E: event that first ball drawn is black

Probability that first ball drawn is black will be $P\left(\frac{E}{S}\right)$

$$\frac{n(E)}{n(S)}$$

where $n(S)$ = no of sample space $= 15$

nE = no of black balls $= 10$

So

Probability that first ball is black $= \frac{10}{15} = \frac{2}{3}$

After removing one ball

no of balls left in urn $= 14$

No of black ball left if first ball drawn is black $= 9$

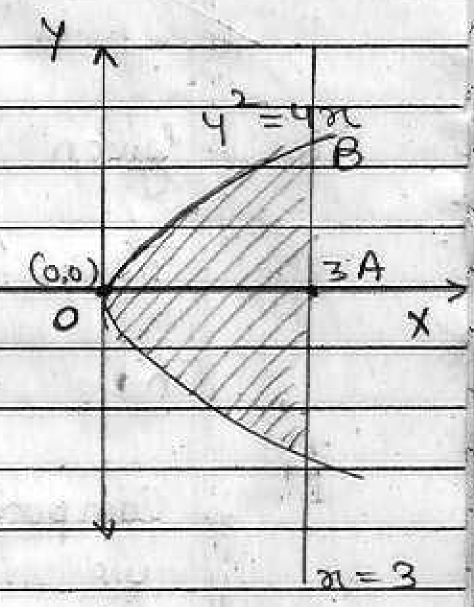
So probability that second ball will be black $= \frac{9}{14}$

Combined probability $= \frac{2}{3} \times \frac{9}{14} = \frac{3}{7}$

Probability that both balls drawn are black $= \frac{3}{7}$

Sol: Q.19

Given



$y^2 = 4x$

$\Rightarrow y = \sqrt{4x} = 2\sqrt{x}$

Area of the shaded region

$= 2 \times$ area of OAB

$= 2 \times \int_0^3 y \, dx$

$= 2 \times \int_0^3 2\sqrt{x} \, dx$

$= 2 \times 2 \times \int_0^3 \sqrt{x} \, dx = 4 \int_0^3 \sqrt{x} \, dx$

$= 4 \left[\frac{x^{\frac{1}{2}+\frac{1}{1}}}{\frac{1}{2}+\frac{1}{1}} \right]_0^3 = 4 \left[\frac{x^{3/2}}{\frac{3}{2}} \right]_0^3$

$= \frac{4 \times 2}{3} \left(x^{3/2} \right)_0^3 = \frac{8}{3} \times 3^{3/2}$

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$\Rightarrow$ Area of shaded region $= 8 \times 3^{\frac{1}{2}}$

$= 8 \times 3^{\frac{1}{2}}$

$= 8\sqrt{3}$ sq. unit

Sol: Q.20 OR

Given lines are

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Comparing with $\vec{r} = \vec{a} + \lambda\vec{b}$ we get

$\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

Shortest distance between two parallel lines is $\left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$

$|\vec{b}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36}$

$= 7$

15

प्रश्न क्र.

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} - 5\hat{k} - (\hat{i} + 2\hat{j} - 4\hat{k})$$

$$= 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$

$$= 2\hat{i} + \hat{j} - \hat{k}$$

~~lets~~ first we will find $\vec{b} \times (\vec{a}_2 - \vec{a}_1)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$\vec{b} \cdot (\vec{a}_2 - \vec{a}_1) = (-3-6)\hat{i} + (2-6)\hat{k} + (12+2)\hat{j}$$

$$= -9\hat{i} - 4\hat{k} + 14\hat{j}$$

$$|\vec{b} \times (\vec{a}_2 - \vec{a}_1)| = \sqrt{(-9)^2 + (-4)^2 + (14)^2}$$

$$= \sqrt{81 + 16 + 196} = \sqrt{293}$$

shortest distance $\dfrac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \dfrac{\sqrt{293}}{7}$

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Sol: Q.21.

To ~~prove~~ solve

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Now solving ~~L.H.S~~

$$\begin{vmatrix} x+a+x+x & x & x \\ x+x+a+x & x+a & x \\ x+x+x+a & x & x+a \end{vmatrix} = 0 \quad \left\{ \text{from operation } C_1 \to C_1+C_2+C_3 \right.$$

$$\Rightarrow \begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix} = 0 \quad \left\{ \text{Taking } (3x+a) \text{ common from } C_1 \right\}$$

$$\Rightarrow (3x+a) \begin{vmatrix} 0 & -a & 0 \\ 0 & a & -a \\ 1 & x & x+a \end{vmatrix} \quad \left\{ \begin{array}{l} \text{from operation} \\ R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{array} \right\}$$

Now expanding along C_1 we get

$$\Rightarrow (3x+a) \times 1 \begin{vmatrix} -a & 0 \\ a & -a \end{vmatrix} = 0$$

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$\Rightarrow (3x + a) \times a^2 = 0$

$\Rightarrow 3x + a = 0$

$\Rightarrow x = -\dfrac{a}{3}$

So the value of x is $-\dfrac{a}{3}$

Sol. Q. 23

$$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx \quad \text{- eqn - (1)}$$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1+\cos^2(\pi - x)} \quad \text{- eqn - (2)}$$

{from property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$}

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} \quad \text{- eqn - (3)}$$

On adding eqn (1) and (3) we get

$$2I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx + \int_0^{\pi} \frac{(\pi - x) \sin x}{1+\cos^2 x}$$

$$= \int_0^{\pi} \frac{(x + \pi - x)\sin x}{1+\cos^2 x}\, dx$$

$$2I = \int_0^{\pi} \frac{\pi \sin x \, dx}{1+\cos^2 x}$$

प्रश्न क्र.

$$\Rightarrow \quad 2I = \pi \int_0 \frac{\sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{\sin x}{1+(1-\sin^2 x)}\,dx$$

$$= \pi \int_0^{\pi} \frac{\sin x}{2-\sin^2 x}\,dx$$

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Sol : Q. 22 OR

Given

$y = \sin(x^2)$

$y' = x^2$

we are to find out $\frac{dy}{dy'}$

Taking $y = \sin \cancel{(x^2)} (x^2)$ and differentiating w.r.t x, we get

$$\frac{dy}{dx} = \cos(x^2) \times 2x \quad - \text{eqn} - ①$$

Similarly ~~tanki~~ taking $y' = x^2$ and differentiating w.r.t x, we get

$$\frac{dy'}{dx} = 2x \quad - \text{eq} - ②$$

Now

$$\frac{dy}{dy'} = \frac{dy}{dx} \div \frac{dy'}{dx}$$

$$\frac{dy}{dy'} = \frac{2x\cos(x^2)}{2x} = \cos(x^2)$$

So differentiation of $\sin(x^2)$ with respect to x^2 is $\cos(x^2)$

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Sol : Q. 17

Given

$Z = 3x + 5y$

Constraints are

$x + 3y \geq 3$, $x + y \geq 2$ $x, y \geq 0$

So, let us first draw the graph for $x + 3y = 3$

Table

x	$y = \frac{3-x}{3}$
0	1
3	0

On putting 0, 0 in $x + 3y \geq 3$, we get 0 which doesn't satisfy the given $x + 3y \geq 3$ so its graph is towards the opposite side of origin

Now let us draw the graph for $x + y \geq 2$

x	$y = 2 - x$
0	2
2	0

On putting 0,0 in $x+y \geq 2$ we get zero which doesn't satisfy $x+y \geq 2$ since $0 < 2$

so its graph will be on the other side of zero

Graph is given ahead

The shaded region is the feasible region and it is unbounded

here corner points are A (3,0), B (0,2) and C

To find the coordinates of C we will have to solve the two equation $x+3y = 3$ and $x+y = 2$

$x+3y = 3$ — eqn - 1
$x+y = 2$ eqn - 2
$\Rightarrow y = 2-x$

Putting this value of y in eqn 1 we get

$x+3(2-x) = 3$
$\Rightarrow x+6-3x = 3$
$\Rightarrow 6-2x = 3 \Rightarrow 6 = 2x+3$
$\Rightarrow x = \frac{3}{2} = 3$

So $y = 2-3/2 = 2-3/2 = +1/2$

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So coordinates of C are $(3, -1)$

Corner point	Objective function $Z = 3x + 5y$
A $(3, 0)$	$Z = 9$
B $(0, 2)$	$Z = 10$
C $\left(\frac{3}{2}, \frac{1}{2}\right)$	$Z = 7$

the minimum value of Z is 7 at coordinates $\left(\frac{3}{2}, \frac{1}{2}\right)$

Pge No. Signature