माध्यमिक शिक्षा मण्डल, मध्यप्रदेश, भोपाल

32 पृष्ठीय

परीक्षार्थी द्वारा भरा जावे ↓

परीक्षार्थी द्वारा भरा जावे →

परीक्षा का विषय	विषय कोड	परीक्षा का माध्यम
Maths	1 5 0	English

[redacted]

उत्तर पुस्तिका का
सरल क्रमांक **C-23** **0140514**

परीक्षार्थी का रोल नम्बर

अंकों में

2	3	6	7	2	5	2	4	2	X

शब्दों में

दो	तीन	छः	सात	दो	पाँच	दो	चार	दो	X

एक	एक	दो	चार	तीन	नौ	पांच	छः	आठ

केन्द्राध्यक्ष / सहायक केन्द्राध्यक्ष एवं पर्यवेक्षक द्वारा भरा जावे →

प्रश्न पत्र का सेट A

क :- परीक्षार्थी का कक्ष क्रमांक 07

ख :- परीक्षा का दिनांक 21/03/2023

परीक्षा का नाम एवं परीक्षा केन्द्र क्रमांक की मुद्रा

H.S.S. केन्द्र क्र. 671001

पर्यवेक्षक का नाम एवं हस्ताक्षर	केन्द्राध्यक्ष / सहायक केन्द्राध्यक्ष के हस्ताक्षर
एम. राठौ	

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे ↓

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे →

प्रमाणित किया जाता है कि होलो क्राफ्ट स्टीकर क्षतिग्रस्त नहीं पाया गया तथा अन्दर के पृष्ठो के अनुरूप मुख्य पृष्ठ पर अंकों की प्रविष्टी एवं अंकों का योग सही है।

निर्धारित मुद्रा : नाम, पदनाम, मोबाईल नम्बर, परीक्षक क्रमांक एवं पदांकित संस्था के नाम की मुद्रा लगाएं।

उप मुख्य परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा	परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा
RAGHWENDRA TIWARI V.No.011120	HJ HARESH SINGH (UMS) GOVT. H.S.S. KANHWARA Mob.No.- 9329482065 V.No.- 10672

केवल परीक्षक द्वारा भरा जावे।
प्रश्न क्रमांक के सम्मुख प्राप्तांको की प्रविष्टी करें।

प्रश्न क्रमांक	पृष्ठ क्रमांक	प्राप्तांक (अंकों में)
1	—	
2	—	
3	—	
4	—	
5	—	
6	—	
7	—	
8	—	
9	—	
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17	—	
18	—	
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26		
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कुल प्राप्तांक		

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Question no. → ①

Choose and write the correct :-

Answers

(i) → (c) f is one-one but not onto

(ii) (b) $-\frac{\pi}{3}$

(iii) (b) $\frac{\pi}{3}$

(iv) (c) 125 |A|

(v) (c) not defined

(vi) (b) ±6

Question no. → ②

fill in the blanks

Answers

(i) $\frac{dy}{dx} = \frac{1}{4} \frac{e^{\sqrt{x}}}{\sqrt{x}\sqrt{e^{\sqrt{x}}}}$

(ii) 10 π cm^2/second

(iii) $\frac{\pi}{2} \cdot x + C$

(iv) Three

(v) Zero

(vi) dc of x axis = (1, 0, 0)
dc of y axis = (0, 1, 0)
dc of z axis = (0, 0, 1)

(vii) $e^{x} \cdot f(x) + c$

Question no. → ③

Match the Column :-

Answers

(i) $\int \tan x \, dx = -\log|\cos x| + c$
(ii) $\int \cot x = \log|\sin x| + c$
(iii) $\int \sec x = \log|\sec x + \tan x| + c$
(iv) $\int \operatorname{cosec} x = \log|\operatorname{cosec} x - \cot x| + c$
(v) $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix} = \cos 2x$
(vi) Derivative of $\sin 2x$ w.r.t x = $2\cos 2x$

Question no. → ④

Answer in one word

[redacted]

Answers :-

(i). A relation which is both empty relation and universal relation present is known as trivial relation. ~~present~~

(ii). → $\frac{\pi}{4}$

(iii). A matrix which has only one ~~Collomb~~ Column is called column matrix. i.e, order = n×1

(iv) $\sqrt{3}$

(v). $P\left(\frac{B}{A}\right) = 0.4$

[redacted]

ii) 0 (zero)

iiy R(x) = 126

Question no. → ⑤

True or false :-

Answers

(i) True

(ii) False

(iii) True

(iv) True

[redacted]

(v) False

(vi) True

Question no. → ⑥ (OR)

$R = \{(1,2)(2,2)(1,1)(4,4)(1,3)$
$(3,3)(3,2)\}$

$A = \{1,2,3,4\}$

Given,

$A = \{1,2,3,4\}$

$R = \{(1,2)(2,2)(1,1)(4,4)(1,3)$
$(3,3)(3,2)\}$

For reflexive relation :-

$aRa \Rightarrow (a,a) \in R$

$R = \{(1,1)(2,2)(3,3)(4,4)\}$

i.e, $(1,1) \in R$
$(2,2) \in R$
$(3,3) \in R$
$(4,4) \in R$

Relation R is satisfied that given element is related to itself.
Thus, R is reflexive relation.

For Symmetric relation :-

$R = \{(1,2)\}$

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$(1,2) \in R \Rightarrow (2,1) \notin R$

i.e,

$aRb \Rightarrow b \not R a$

$(a,b) \in R \Rightarrow (b,a) \notin R$

Here, (a,b) is related to R
but (b,a) is not related to R
Thus, R is not Symmetric relation.

For transitive relation :-

$R = \{(1,3)\ (3,2)\ (1,2)\}$

$(1,3) \in R$ and $(3,2) \in R \Rightarrow (1,2) \in R$

aRb and $bRc \Rightarrow aRc$

here, Above relation (1,3) is element of R and (3,2) is element of r then (1,2) is also the element of R.
Thus, R is transitive relation.

Question no. → ⑦ (OR)

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ — ①

$x \in [-1, 1]$

The domain of $\sin^{-1}x$ is :-

$-1 \le x \le 1$

i.e., $x \in [-1, 1]$

let:

$\sin^{-1}x = \theta$

$x = \sin\theta$

m ①, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\theta + \cos^{-1}x = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} - \cos^{-1}x$

, $\theta = \sin^{-1}x$

$\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$

$\frac{\pi}{2} = \sin^{-1}x + \cos^{-1}x$

Thus, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ at $x \in [-1,1]$

→ proved.

Question no. → ⑧

$A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

find $(A+B)'$

$A+B = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$$A+B = \begin{bmatrix} -2+(-1) & 3+0 \\ 1+1 & 2+2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} -2-1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} -3 & 3 \\ 2 & 4 \end{bmatrix}$$

[redacted]

$$(A+B)' = \begin{bmatrix} -3 & 2 \\ 3 & 4 \end{bmatrix}$$

→ The Answer is $(A+B)' = \begin{bmatrix} -3 & 2 \\ 3 & 4 \end{bmatrix}$

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Question no. → ⑨ (OR)

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\frac{dy}{dx} = ?$$

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

let $x = \tan\theta$

$\theta = \tan^{-1} x$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$y = \sin^{-1}(\sin 2\theta)$$

$$\left[\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$$y = Sin^{-1}(Sin 2\theta)$$

$$y = 2\theta$$

$$y = 2\tan^{-1} x \quad [\because Sin^{-1}(Sin x) = x]$$

Differentiation w.r.t x

$$\frac{dy}{dx} = 2 \cdot \frac{d}{dx} \tan^{-1} x$$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$ → <u>Ans</u>

<u>Question no.</u> → ⑩

$$f(x) = 12x - 3$$

let $x_1, x_2 \in R$

again, $x_1 = x_2$

$$12x_1 = 12x_2$$

$$12x_1 - 3 = 12x_2 - 3$$

$$12x_1 - 3 + 3 = 12x_2 - 3 + 3$$

$$12x_1 = 12x_2$$

$$x_1 = x_2$$

$$f(x_1) = f(x_2)$$

The given function is positive for all value of x.
So, R is increasing for all value of x.
Thus, $f(x)$ is increasing function on R.

<u>hence</u> proved.

[redacted]

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प्रश्न क्र.

Question no. → ⑪

$$\int_{-1}^{1} \sin^5 x \cos^4 x \, dx$$

$$I = \int_{-1}^{1} \sin^5 x \cos^4 x \, dx$$

$$I = \int_{-1}^{1} \sin^4 x \cdot \sin x \cos^4 x \, dx$$

$$I = \int_{-1}^{1} (1-\cos^2 x)^2 \sin x \cos^4 x \, dx$$

$$[\because \sin^2 x = 1-\cos^2 x]$$

let $\cos x = t$ — ①

$$-\sin x \, dx = dt$$

$$\sin x \, dx = -dt$$

$$I = -\int_{-1}^{1} (1-t^2)^2 t^4 \, dt$$

$$I = -\int_{-1}^{1} (1+t^4-2t^2) t^4 \, dt$$

$$I = -\int_{-1}^{+1} t^4+t^8-t^6 \, dt$$

$$I = -\left[\frac{t^5}{5}+\frac{t^9}{9}-\frac{t^7}{7}\right]_{-1}^{+1}$$

$$I = -\left[\frac{\cos^5 x}{5}+\frac{\cos^9 x}{9}+\frac{\cos^7 x}{7}\right]_{-1}^{1}$$

From eqn ①

$$\cos x = t$$

Change the limit

$$\cos x = 1$$

$$x = 0°$$

$\cos x = -1$

$x = \pi$

The new limit is :-

$$I = -\left[\frac{\cos^5 x}{5} + \frac{\cos^9 x}{9} - \frac{\cos^7 x}{7}\right]_{\pi}^{0}$$

$$I = -\left[\left(\frac{\cos^5 0^\circ}{5} + \frac{\cos^9 0}{9} - \frac{\cos^7 0^\circ}{7}\right) - \left(\frac{\cos^5 \pi}{5} - \frac{\cos^9 \pi}{9} + \frac{\cos^7 \pi}{7}\right)\right]$$

$$I = -\left[\left(\frac{1}{5} + \frac{1}{9} - \frac{1}{7}\right) - \left(-\frac{1}{5}\right) - \left(-\frac{1}{9}\right) + \left(-\frac{1}{7}\right)\right]$$

$$I = -\left[\frac{1}{5} + \frac{1}{9} - \frac{1}{7} + \frac{1}{5} + \frac{1}{9} - \frac{1}{7}\right]$$

$$I = -\left[\frac{2(63+35-45)}{315}\right]$$

$$I = -\left[\frac{126+70-90}{315}\right]$$

$$I = -\left[\frac{106}{315}\right]$$

$$I = -\frac{106}{315}$$

Ans :- $I = -\frac{106}{315}$

प्रश्न क्र.

Question no. → ⑫

$\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{b} = 2\hat{i}+\hat{k}$

Projection of vector $\vec{a}$ on vector $\vec{b}$

$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\vec{a}\cdot\vec{b} = (\hat{i}+2\hat{j}+3\hat{k})\cdot(2\hat{i}+\hat{k})$

$\vec{a}\cdot\vec{b} = (2+0+3)$

$\vec{a}\cdot\vec{b} = 5$

$|\vec{b}| = \sqrt{2^2+1^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$

Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{5}{\sqrt{5}}$

$= \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5}$

$= \sqrt{5}$

Projection of $\vec{a}$ on $\vec{b}$ is $\sqrt{5}$

Question no. → ⑬ (OR)

$\vec{a} = x(\hat{i}+\hat{j}+\hat{k})$

Given

$\vec{a} = x(\hat{i}+\hat{j}+\hat{k})$

$\vec{a}$ is unit vector

i.e, $|\vec{a}| = 1$.

$\vec{a} = x(\hat{i}+\hat{j}+\hat{k})$

$|\vec{a}| = x\hat{i}+x\hat{j}+\hat{k}$

$1 = \sqrt{x^2+x^2+x^2}$

$1 = \sqrt{3x^2}$

$1 = \sqrt{3}\,x$

$x = \pm\frac{1}{\sqrt{3}}$

The value of $x = \pm\frac{1}{\sqrt{3}}$

Question no. → ⑭

Given,

Direction ratios of a line are $(-18, 12, -4)$

i.e,

$a = -18$

$b = 12$

$c = -4$

$A = (-18, 12, -4)$

Direction cosines :-

$x = \frac{\bar{a}}{\sqrt{a^2+b^2+c^2}}$, $y = \frac{b}{\sqrt{a^2+b^2+c^2}}$

$z = \frac{c}{\sqrt{a^2+b^2+c^2}}$

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$$x = \frac{-18}{\sqrt{(18)^2+(12)^2+(-4)^2}}$$

$$x = \frac{-18}{\sqrt{324+144+16}}$$

$$x = \frac{-18}{\sqrt{484}}$$

$$x = \frac{\cancel{-18}}{\cancel{22}} \frac{-9}{11}$$

$$x = \frac{-9}{11}$$

$$y = \frac{12}{\sqrt{484}}$$

$$y = \frac{\cancel{12}}{\cancel{22}} \frac{6}{11}$$

$$y = \frac{6}{11}$$

$$z = \frac{-4}{\sqrt{484}}$$

$$z = \frac{\cancel{-4}}{\cancel{22}} \frac{-2}{11}$$

$$z = \frac{-2}{11}$$

$$(x, y, z) = \left(\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}\right)$$

The direction Cosines of the vector :-

$$\left(\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}\right)$$

⑥
18
×18
①
144
18×
①
324
144
16
484

B
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2
2
14
4×
84

Question no. → ⑮ (OR)

$f(x) = x^2 - 4x + 6$

Given,

$f(x) = x^2 - 4x + 6$

differentiation w.r.t x

$f'(x) = 2x - 4$

for Increasing and decreasing :-

let, $f'(x) = 0$

$2x - 4 = 0$

$2x = 4$

$x = \frac{4}{2}$

$x = 2.$

The interval is $(-\infty, 2)$ $(2, +\infty)$

Test value

Interval	Value	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, 2)$	1	-ve.	Decreasing
$(2, +\infty)$	3	+ve	Increasing

at $x = 1$,
$f(x) = 2(1) - 4 = 2 - 4 = -2$

at $x = 3$, $f(x) = 2(3) - 4 = 6 - 4 = 2$

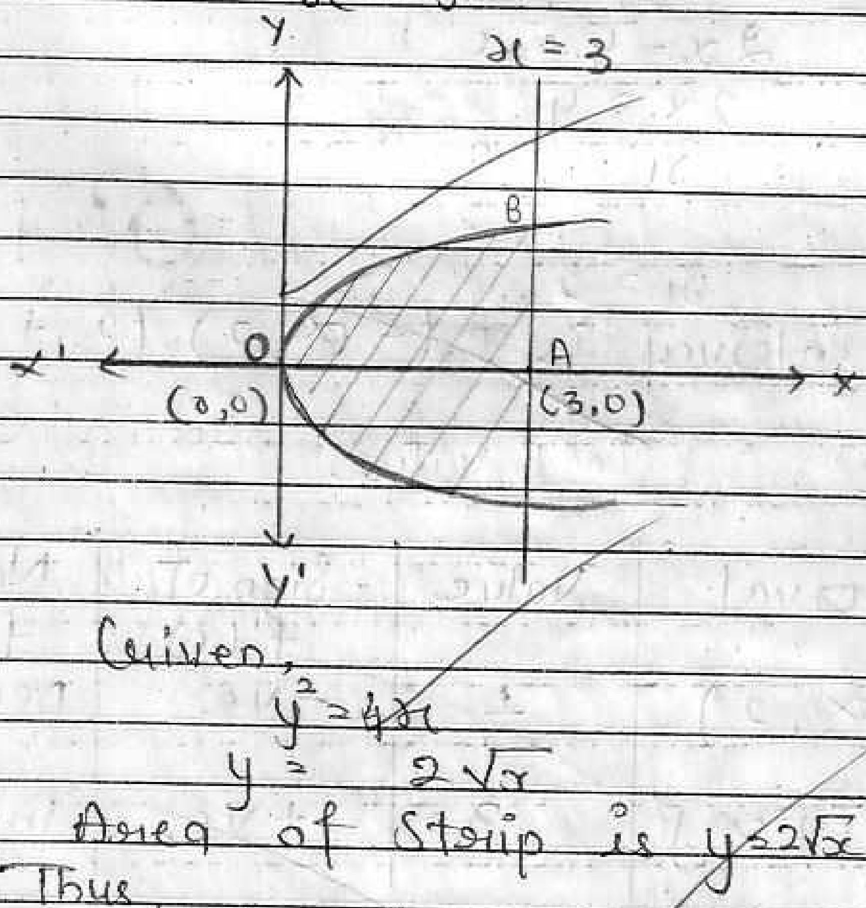
It is cleared that $f(x) = x^2-4x+6$ is decreasing at interval $(-\infty, 2)$

Ans :- Decreasing function at $(-\infty, 2)$

Question no. → (16) (OP)

$y^2 = 4x$

$x = 3$



Given,

$y^2 = 4x$

$y = 2\sqrt{x}$

Area of Strip is $y = 2\sqrt{x}$

Thus,

Required area of the region :-

□AOB = ?

Area $= 2 \times \int_0^3 y \, dx$

$=$

$= 2 \times \int_0^3 2\sqrt{x} \, dx$

$= 4 \int_0^3 \sqrt{x} \, dx$

$= 4 \int_0^3 x^{1/2} \, dx$

$= 4 \int_0 \left[\frac{x^{1/2+1}}{1/2+1} \right]_0^3$

$= 4 \int_0 \left[\frac{x^{3/2}}{3/2} \right]_0^3$

$= 4 \cdot \frac{2}{3} \left[x^{3/2} \right]_0^3$

$= \frac{8}{3} \left[3^{3/2} - 0 \right]$

$\frac{8}{3} \left[3\sqrt{3} \right]$

$= 8\sqrt{3}$ square units

The area of the region is $8\sqrt{3}$ sq. units.

P.T.O.

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Question No. → ⑰ (OP)

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

Given

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}c$$

$$\tan^{-1}y - \tan^{-1}x = \tan^{-1}c$$

$$\tan^{-1}\frac{y-x}{1+yx} = \tan^{-1}c$$

$$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}\right]$$

Formula used :-

$$\tan^{-1}x \rightarrow \frac{1}{1+x^2}$$

$$\tan^{-1}y \rightarrow \frac{1}{1+y^2}$$

$$\tan^{-1}\frac{y-x}{1+yx} = \tan^{-1}c$$

$$\frac{y-x}{1+yx} = \tan(\tan^{-1}c)$$

$$\frac{y-x}{1+yx} = C$$

$$C = \frac{y-x}{1+yx}$$

Solution of given diff equation is :-

$$C = \frac{y-x}{1+yx}$$

Question no. → ⑱

$Z = 3x + 5y$

$x + 3y \geq 3$ — ①

$x + y \geq 2$ — ②

$x, y \geq 0$ — ③

Given,

$Z = 3x + 5y$

$x + 3y \geq 3$ — ①

$x + y \geq 2$ — ②

$x, y \geq 0$ — ③

First Change equation ① inequality to equality :-

$x + 3y = 3$

$3y = 3 - x$

$$y = \frac{3-x}{3}$$

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x	0	3 (3)
y	1	0 (0)

The point (0,0) is put in eqn ①

$x+3y \geq 3$

$0 \geq 3$

It is false Statement.

The equation ① is not in feasionable region.

From eqn. ② :

$x+y \geq 2$

Change into equation

$x+y = 2$

$y = 2-x$

x	0	2
y	2	0

(0,0) is put in inequality ②

$x+y \geq 2$

$0 \geq 2$

It is false Statement

It is not belong to feasonable region.

equation ③,

$x, y \geq 0$

It is cleared that feasonable region is in first quadrant.

From the graph.

It is cleared that the feasionable region is above both the inequality. (Not defined) that is :-

A and B is below the feasionable region.

A (0,2)
B (3,0)
O (0,0)

at O (0,0)

$Z = 3x + 5y$

$Z = 3(0) + 5(0)$

$\boxed{Z = 0}$ (~~me~~ minimize)

at A (0,2)

$Z = 3(0) + 5(2)$

$Z = 0 + 10$

$\boxed{Z = 10}$

at B (3,0)

$Z = 3(3) + 5(0)$

$Z = 9 + 0$

$\boxed{Z = 9}$

The constraints is minimize at O (0,0) and minimum value is Zero (0).

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Question no → (20) (OR)

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from R_1

$$(a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_1$

$$= (a+b+c) \times 1 \begin{vmatrix} -b-c-a & 0 \\ 0 & -c-a-b \end{vmatrix}$$

[Expanding the determinant]

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[redacted]

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$= (a+b+c)\,(-b-c-a)\,(-c-a-b)$

$= (a+b+c)\,(-b+c+a)^2$

$= (a+b+c)^3 = RHS$

$LHS = RHS$

Question no → ㉑ (OR)

Diff $\sin x^2$ w.r.t x^2

let $\sin x^2 = y$

~~$x^2 = y$~~ $x^2 = x$

diff w.r.t x

$x = \sin x^2$

$\frac{dy}{d\theta} = \frac{d}{d\theta} \sin x^2$

$\frac{dy}{d\theta} = \cos x^2 \frac{d}{d\theta} x^2$

$= \cos x^2 \; 2x$ — ①

~~$y = x^2$~~

~~$\frac{dy}{d\theta}$~~ $x = x^2$

$\frac{dx}{d\theta} = \frac{d}{d\theta} x^2$

$\frac{dx}{d\theta} = 2x$ — ②

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

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from ① and ②

$$\frac{dy}{dx} = \frac{\cancel{2x}\,\cos x^2}{\cancel{2x}}$$

$$\frac{dy}{dx} = \cos x^2$$

Answer :- $\frac{dy}{dx} = \cos x^2$

~~Question no~~ → ~~㉒~~ ~~(OR)~~

~~$\int_0^{\pi/2} \frac{\sec^5 x}{\sin^5 x + \cos^5 x} dx$~~

Question no → ⑲

According to the Question
Total no of cards n(S) = 10

let
E = be event have even number
F = be event have more than three

E = {2, 4, 6, 8, 10}
n(E) = 5

F = {4, 5, 6, 7, 8, 9, 10}
n(F) = 7

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$(E \cap F) = \{4, 6, 8, 10\}$

$n(E \cap F) = 4$

According to the Question :-

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{\frac{n(E \cap F)}{n(S)}}{\frac{n(F)}{n(S)}}$$

$$P\left(\frac{E}{F}\right) = \frac{\frac{4}{10}}{\frac{7}{10}}$$

$$= \frac{\cancel{40}}{\cancel{70}} = \frac{4}{7}$$

$$P\left(\frac{E}{F}\right) = \frac{4}{7}$$

The probability that it is even number is $\frac{4}{7}$.

Ans :- $P\left(\frac{E}{F}\right) = \frac{4}{7}$

प्रश्न क्र.

B
S
E

Question no. → (23) (OR)

$\vec{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$
$\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$

Given,
$\vec{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$
$\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$

S.D. = ?
From $\vec{r}_1$ and $\vec{r}_2$:-

$\vec{a}_1 = \hat{i}+2\hat{j}-4\hat{k}$
$\vec{a}_2 = 3\hat{i}+3\hat{j}-5\hat{k}$

$\vec{b} = (2\hat{i}+3\hat{j}+6\hat{k})$

$$S.D. = \left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$$

$(\vec{a}_2-\vec{a}_1) = (3\hat{i}+3\hat{j}-5\hat{k}) - (\hat{i}+2\hat{j}-4\hat{k})$
$\vec{a}_2-\vec{a}_1 = 2\hat{i}+\hat{j}-\hat{k}$

$|\vec{b}| = \sqrt{2^2+3^2+6^2}$
$= \sqrt{4+9+36}$
$= \sqrt{49}$

$|\vec{b}| = 7$

~~SD~~ Now,

$(\vec{a_2}-\vec{a_1}) \times \vec{b} = (2\hat{i}+\hat{j}-\hat{k}) \times (2\hat{i}+3\hat{j}+6\hat{k})$

$$(\vec{a_2}-\vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$

$= 9\hat{i} - 14\hat{j} + 4\hat{k}$

$|(\vec{a_2}-\vec{a_1}) \times \vec{b}) = \sqrt{9^2+(-14)^2+4^2}$

$= \sqrt{81+196+16}$

$= \sqrt{81+212}$

$= \sqrt{293}$

$$SD = \left| \frac{(\vec{a_2}-a_1) \times \vec{b}}{|\vec{b}|} \right|$$

$$SD = \frac{\sqrt{293}}{7}$$

The shortest distance of the parallel lines are :-

$$S.D. = \frac{\sqrt{293}}{7}.$$

प्रश्न क.

B
S
E

Question no. → (22)

$$\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$$

$$[\because \sin(\pi-x) = \sin x]$$

$$I = \int_0^\pi \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$$

$$I = \int_0^\pi \left[\frac{\pi\sin x}{1+\cos^2 x} - \frac{x\sin x}{1+\cos^2 x}\right]dx$$

$$I = \int_0^\pi \frac{\pi\sin x\,dx}{1+\cos^2 x} - \int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$\left[\because \int_0^\pi \frac{x\sin x}{1+\cos^2 x} = I\right]$$

क्र.

$$I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} - I$$

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}$$

let $\cos x = t$

$-\sin x\,dx = dt$

$$\left[\because \cos\pi = -1,\ \cos 0 = 1\right]$$

$$2I = \pi \int_1^{-1} \frac{-1}{1+t^2}\,dt$$

$$2I = -\pi \int_1^{-1} \frac{1}{1+t^2}\,dt$$

$$\left[\int \frac{1}{1+x^2}\,dx = \tan^{-1} x\right]$$

$$2I = -\pi \left[\tan^{-1} t\right]_1^{-1}$$

$$2I = -\pi \left[\tan^{-1} 1 - \tan^{-1} 1\right]$$

$$2I = -\pi \left[\tan^{-1}\left(-\tan\frac{\pi}{4}\right) - \tan^{-1}\left(\tan\frac{\pi}{4}\right)\right]$$

प्रश्न क्र.

$$2I = -\pi\left[-\frac{\pi}{4} - \frac{\pi}{4}\right]$$

$$2I = -\pi - \frac{2\pi}{4}$$

$$2I = -\pi - \frac{\pi}{2}$$

$$2I = -\pi\left(-\frac{\pi}{2}\right)$$

$$2I = \frac{\pi^2}{2}$$

$$I = \frac{\pi^2}{4}$$

B
S
E

$$I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx = \frac{\pi^2}{4}$$

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