केवल मूल्यांकनकर्ता के उपयोग हेतु!

माध्यमिक शिक्षा मण्डल, मध्यप्रदेश, भोपाल

32 पृष्ठीय

केवल परीक्षक द्वारा भरा जावे। प्रश्न क्रमांक के सम्मुख प्राप्तांकों की

प्रश्न क्रमांक	पृष्ठ क्रमांक	...ांक (अंकों में)	प्रश्न क्रमांक	पृष्ठ क्रमांक	...ांक (अंकों में)
1			17		
2			18		
3			19		
4			20		
5			21		
6			22		
7			23		
8			24		
9			25		
10			26		
11			27		
12			28		
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16					

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे

प्रमाणित किया जाता है कि अन्दर के पृष्ठों के अनुरूप मुख्य पृष्ठ पर अंकों की प्रविष्टी एवं अंकों का योग सही है।

निर्धारित मुद्राः नाम, पदनाम, मोबाईल नम्बर, परीक्षक क्रमांक एवं पदांकित संस्था के नाम की मुद्रा लगाएं।

उप मुख्य परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा

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परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा

श्रीमती निर्मला मुनिया
उच्च मा. शिक्षक
शा.मॉ.उ.मा.वि.थांदला
प.क्रमांक. HSSMAT24003

② योग पूर्व पृष्ठ + पृष्ठ 2 के अंक = कुल अंक

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Question - 1

(i) zero (0).

(ii) $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25} = 0.64$ Ans

(iii) Scalar matrix: A square matrix in which all the diagonal elements are constant (or scalar) and equal, and except diagonal elements all other are zero

eg. $A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ k is scalar

(iv) 2 (two) Ans : $\hat{i}\times\hat{j} = \hat{k}$ and $\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\cdot\hat{k} + \hat{i}\cdot\hat{i} = 1+1 = 2$

(v) -2.

$|adj(A)| = |A|^{n-1} = |A|^{2-1} = |A| = 4-6 = -2$

(n is order).

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(vi) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(vii) $f(x)$ is increasing in interval $[1, \infty)$

as $f'(x) = 2x-2$

$= 2(x-1) \geq 0$

it is positive when $x \in [1, \infty)$

<u>Question : 2</u>

(i) (a) f is one-one onto

(ii) (b) $\frac{3\pi}{4}$

(iii) (d) $\frac{\pi}{3}$

(iv) (c) $\pm\sqrt{3}$

(v) (a) $\theta = \frac{\pi}{2}$

(vi) (c) $\frac{1}{18}$

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Question . 3

(i) $\frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x}$

(ii) $\frac{dA}{dr} = 2\pi r = 2\pi(6) = 12\pi \ cm^2/cm$

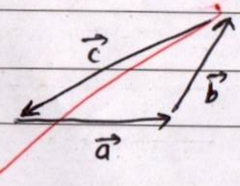
(iii) $\sin(\sin^{-1}x) = x$

(iv) $l^2 + m^2 + n^2 = 1$

(v) two (2) $R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

$R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2)\}$

(vi) zero.



$\vec{a} + \vec{b} + \vec{c} = 0$.

5

याग पूव पृष्ठ + पृष्ठ 5 के अंक = कुल अंक

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Question - 4

(i) True

(ii) False

(iii) True.

(iv) False

(v) False

(vi) True.

P.T.O.

6 + पृष्ठ 6 का अंक =

माध्यमिक शिक्षा मण्डल मध्यप्रदेश

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Question. 5

(i) $\int \frac{dx}{\sqrt{x^2+a^2}}$ = (f) $\log|x+\sqrt{x^2+a^2}|+c$

(ii) $\int \sqrt{x^2+a^2}\,dx$ = (e) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+c$

(iii) $\int \tan x\,dx$ = (g) $\log|\sec x|+c$

(iv) $\int \frac{1}{\sqrt{x^2-9}}dx$ = ~~(b) $\frac{x}{2}\sqrt{x^2-9}$~~ (d) $\log|x+\sqrt{x^2-9}|+c$

(v) $\int \sqrt{x^2-9}\,dx$ = (b) $\frac{x}{2}\sqrt{x^2-9}-\frac{9}{2}\log|x+\sqrt{x^2-9}|+c$

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Questions

(i) $\int \frac{dx}{\sqrt{x^2+a^2}}$ (f) $\log |x + \sqrt{x^2+a^2}| + c$

(ii) $\int \sqrt{x^2+a^2}\, dx$ (e) $\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log |x + \sqrt{x^2+a^2}| + c$

(iii) $\int \tan x\, dx$ (g) $\log |\sec x| + c$

(iv) $\int \cot x\, dx$ (a) $\log |\sin x| + c$

(v) $\int \sqrt{x^2-9}\, dx$ (b) $\frac{x}{2}\sqrt{x^2-9} - \frac{9}{2}\log |x + \sqrt{x^2-9}| + c$

(vi) $\int \frac{1}{\sqrt{x^2-9}}\, dx$ (d) $\log |x + \sqrt{x^2-9}| + c$

(vii) Simplest form of $\tan\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ (c) $\operatorname{cosec}^{-1} x$.

8 + =

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Question. 6

Given: $\vec{a} = \hat{i}+\hat{j}-\hat{k}$ $\quad \vec{b} = \hat{i}-\hat{j}+\hat{k}$

To find: angle between $\vec{a}$ & $\vec{b}$ (θ)

we know that

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{(\hat{i}+\hat{j}-\hat{k})\cdot(\hat{i}-\hat{j}+\hat{k})}{\sqrt{1^2+1^2+1^2}\,\sqrt{1^2+1^2+1^2}} = \frac{1-1-1}{\sqrt{3}\,\sqrt{3}} = \frac{-1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{1}{3}\right) = \pi - \cos^{-1}\left(\frac{1}{3}\right)$$ Ans $\boxed{\theta = \pi - \cos^{-1}\left(\frac{1}{3}\right)}$ → obtuse angle

or acute angle is $\boxed{\theta_1 = \cos^{-1}\left(\frac{1}{3}\right)}$

Question. 7 OR

Given: line $l_1 \Rightarrow \frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $l_2 \Rightarrow \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

To prove: $l_1 \perp l_2$.

the direction ratios of $l_1 = (7,-5,1)$ let it be (a_1, b_1, c_1)

and $l_2 = (1,2,3)$ and (a_2, b_2, c_2)

angle between lines:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} = \frac{7\times1 + (-5)(2) + (1)(3)}{\sqrt{7^2+5^2+1^2}\sqrt{1^2+2^2+3^2}} = \frac{7-10+3}{\sqrt{7^2+5^2+1^2}\sqrt{1^2+2^2+3^2}}$$

$$= 0$$

since $\cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

Hence proved that $l_1 \perp l_2$.

Question. 8

Given: Relation $R = \{(1,2),(2,1)\}$ in set $A = \{1,2,3\}$ (say)

To prove: R is symmetric but not reflexive and not transitive.

since: $(a,b) \in R \Rightarrow (b,a) \in R$

$(1,2) \in R \Rightarrow (2,1) \in R$

Therefore R is symmetric

now, for Reflexive $(a,a) \in R \ \forall \ a \in A$

but it doesnot contain $\{(1,1),(2,2),(3,3)\}$

So, it is not Reflexive

for Transitive if $(a,b) \in R$ and $(b,c) \in R \Longrightarrow (a,c) \in R$

Here $(1,2) \in R$ and $(2,1) \in R \Rightarrow (1,1) \notin R$

It does not contain $(1,1)$.

Hence It is not transitive.

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Question : 9

To prove: $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$ — ①
in RHS

RHS = $\cos^{-1}(4x^3 - 3x)$

RHS = $\cos^{-1}(4\cos^3\theta - 3\cos\theta)$

RHS = $\cos^{-1}(\cos 3\theta)$ Because $(\cos 3\theta = 4\cos^3\theta - 3\cos\theta)$

RHS = 3θ As $\{\cos^{-1}\cos x = x\}$

from ①

RHS = $3\cos^{-1}x$ = LHS

Since RHS = LHS

Hence proved.

Question : 10

Given: $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$ — ①

To find: x and y

(11) 40 + 2 = 42

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from eqn (1)

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$ (multiplication of scalar x and y)

$$\Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$ (Adding the matrices)

since both the matrices are equal, then their corresponding elements are also equal

so

$2x - y = 10$ —— (2)

$3x + y = 5$ —— (3)

Add (2) & (3) eqn, we get

$5x = 15$

$\boxed{x = 3}$ put in (3)

$9 + y = 5$

$\boxed{y = -4}$

Hence required value are of $\boxed{x = 3}$ and $\boxed{y = -4}$ Ans

Question 11 'OR'

Given: vertices of a triangle, let $(x_1, y_1) = (1, 0)$
$(x_2, y_2) = (6, 0)$
$(x_3, y_3) = (4, 3)$

To find: Area of triangle

we know that, area (A) of triangle is given by

$$A = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right| = \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

expanding the determinant along row 1

$$A = \frac{1}{2}\left[1(0-3) - 0 + 1(6 \times 3 - 0)\right]$$

$$= \frac{1}{2}\left[-3 + 18\right] = \frac{15}{2} \text{ square units}$$

Hence Area of the triangle is $\frac{15}{2}$ square units.

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Question 12 'OR'

Given: $x = a\cos\theta$ — ①, $y = b\sin\theta$ — ②

differentiate eqn ① w.r.t θ

$$\frac{dx}{d\theta} = a.(-\sin\theta)$$

and on differentiating eqn ② w.r.t θ, we get

$$\frac{dy}{d\theta} = b\cos\theta$$

using parametric differentiation

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\cos\theta}{a(-\sin\theta)} = -\frac{b}{a}\cot\theta \quad \text{Ans}$$

2

Question 13

Let $I = \int \cos^2 x \, dx$

$$= \int \left(\frac{1+\cos 2x}{2}\right) dx \qquad \text{as } \cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \frac{1}{2}\int (1+\cos 2x)\, dx$$

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So $I = \frac{1}{2}\left[\int 1\,dx + \int \cos 2x\,dx\right]$

$I = \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c$ $\left(\text{as } \int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{b} + c\right)$

So, $I = \frac{x}{2} + \frac{1}{4}\sin 2x + c$ Ans

Question: 14

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Given: $y = e^{-3x}$

differentiate w.r.t x

$\frac{dy}{dx} = e^{-3x}(-3)$ —— (1)

now, double differentiate w.r.t x

$\frac{d^2y}{dx^2} = (-3)e^{-3x}(-3) = 9e^{-3x}$ —— (2)

Add eqn (1) and (2) and subtract $6y$

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = (-3)e^{-3x} + 9e^{-3x} - 6e^{-3x}$

$= +6e^{-3x} - 6e^{-3x}$

$= 0.$

Hence verified.

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Question : 15 'OR'

Given: $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

To find: projection $\vec{a}$ on $\vec{b}$.

we now, that projection p of vector $\vec{a}$ on $\vec{b}$ is given by:

$$p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \quad \text{--- (1)}$$

now, $\vec{a} \cdot \vec{b} = (2\hat{i} + 3\hat{j} + 2\hat{k})(\hat{i} + 2\hat{j} + \hat{k})$

$= 2 + 6 + 2 = 10$

and $|\vec{b}| = \sqrt{1^2 + 2^2 + 1} = \sqrt{6}$

put in (1)

$$\boxed{p = \frac{10}{\sqrt{6}}} \text{ Ans.}$$

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It is required projection.

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Question : 16

Suppose bag : A contains 3 Red and 3 Black balls and bag : B contains 4 Red and 5 black balls.

Let E_1 : be the event of drawing a ball from bag A
E_2 : be the event of drawing ball from B
and
A : be the event of drawing red ball.

now, we need to find ~~P(E/E1)~~ $P\left(\frac{E_1}{A}\right) = ?$

$P(E_1) = \frac{1}{2}$ (Bag is selected randomly)

$P(E_2) = \frac{1}{2}$

and $P\left(\frac{A}{E_1}\right) = \frac{3}{6} = \frac{1}{2}$ (As bag A contain 3 Red and total 6 balls)

$P\left(\frac{A}{E_2}\right) = \frac{4}{9}$ (As bag B contains 4 Red and total 9 balls)

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using the Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{6}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{9}\right)}$$ (putting the values)

$$= \frac{\left(\frac{3}{6}\right)}{\left(\frac{3}{6}\right) + \left(\frac{4}{9}\right)} = \frac{\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right) + \left(\frac{4}{3}\right)} = \frac{\left(\frac{3}{2}\right)}{\frac{9+8}{6}} = \frac{\left(\frac{3}{2}\right)}{\left(\frac{17}{6}\right)}$$

$$= \frac{3}{2} \times \frac{6}{17} = \frac{9}{17}$$

Hence required probability

$$\boxed{P\left(\frac{E_1}{A}\right) = \frac{9}{17}}$$ Ans

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Question 17 'OR'

Given: $f(x) = 4x^3 - 6x^2 - 72x + 30$ —— (1)

To find: Interval of (a) increasing (b) decreasing

First, we find derivative of $f(x)$ i.e. $f'(x)$

on differentiating eqn (1) w.r.t x

$$f'(x) = 12x^2 - 12x - 72 + 0$$

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(a) for, $f(x)$ to be increasing function

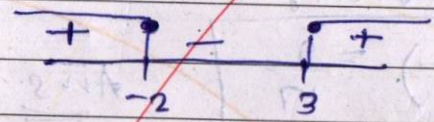
$$f'(x) \geq 0$$

so, $\Rightarrow 12x^2 - 12x - 72 \geq 0$

$\Rightarrow x^2 - x - 6 \geq 0$

$\Rightarrow (x-3)(x+2) \geq 0$

plotting on wavy curve, we get



so, for $f(x)$ to be increasing,

$\boxed{x \in (-\infty, -2] \cup [3, \infty)}$ → Ans (a).

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(b) we can see that $f'(x) \leq 0$ in interval $x \in [-2, 3]$

also for decreasing function

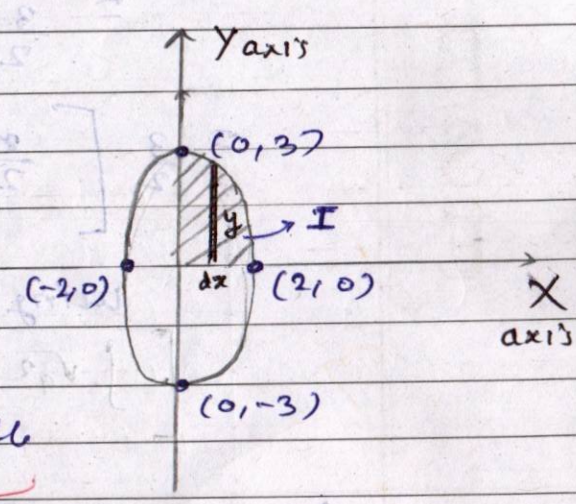
$f'(x) \leq 0$

Hence for decreasing function

$\boxed{x \in [-2, 3]}$ — Ans(b).

Question : 18 OR

To find : Area (A) enclosed by the ellipse : $\frac{x^2}{4} + \frac{y^2}{9} = 1$



Suppose area enclosed in first quadrant is I, then $A = 4I$ — ①

Because ellipse is symmetric in all four quadrant,

Now $I = \int y\,dx$

So, from eq^n of ellipse

$\frac{y^2}{9} = 1 - \frac{x^2}{4} \Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}}$

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$$\cancel{I = \int 3\sqrt{\frac{x^2}{4} - 1}}$$

$$I = \int_0^2 3\sqrt{1 - \frac{x^2}{4}}$$ limits from $x=0$ to $x=2$

$$= 3\int_0^2 \sqrt{\frac{4-x^2}{4}}$$

$$= \frac{3}{2}\int_0^2 \sqrt{4-x^2}$$

$$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

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using the formula

$$\left[\int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{3}{2}\left[\left(\frac{2}{2}\sqrt{4-4} + \frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right)\right) - \left(\frac{0}{2}\sqrt{4-0^2} + \frac{4}{2}\sin^{-1}\left(\frac{0}{2}\right)\right)\right]$$

$$I = \frac{3}{2}\left[\left(0 + 2\sin^{-1}(1) - 0\right)\right]$$

$$I = \frac{3}{2}(2)\left(\frac{\pi}{2}\right) \qquad \left(\text{as } \sin^{-1}(1) = \frac{\pi}{2}\right]$$

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so, $I = \frac{3\pi}{2}$

Now put I in eqn (1)

$A = 4I$

$= 4\left(\frac{3\pi}{2}\right) = 6\pi$ square units.

Hence the area enclosed by ellispce is <u>6π square units</u> Ans.

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<u>Question 19</u>

Given: Objective function $z = 3x + 4y$

subject to the constranits $x + y \leq 4$

$x \geq 0,\ y \geq 0$

now, we will plot the given inequality on graph paper (page: 33) using some point. to find feasible region.

for inequality $x + y \leq 4$

corresponding equation is: $x + y = 4$

putting $y = 0$, $x = 4$

& at $x = 0$, $y = 4$

x	4	0
y	0	4

let point A be (4,0)
and B be (0,4) O is origin (0,0).

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put (0, 0) in $x + y \leq 4$

we get $0 \leq 4$

which is true, hence feasible region lies towards origin. and

from $x \geq 0$, $y \geq 0$ represents first quadrant.

from graph the feasible region in OAB

now, we will use the corner point method to maximize $z = 3x + 4y$ (Because feasible region is bounded)

corner point on graph OAB	corresponding value of objective function $z = 3x + 4y$
O (0, 0)	$z = 0$
A (4, 0)	$z = 12$
B (0, 4)	$\boxed{z = 16}$ → maximum

Hence maximum value of z is <u>16</u> at point (0, 4).

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Question 20 OR

Given: $r = \hat{i}+2\hat{j}+\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k}$ say line l_1

and $r = 2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$ and line l_2

comparing l_1 & l_2 from general form $r = \vec{a}+\lambda\vec{b}$

we get

$\vec{a_1} = \hat{i}+2\hat{j}+\hat{k}$ and $\vec{b_1} = \hat{i}-\hat{j}+\hat{k}$

$\vec{a_2} = 2\hat{i}-\hat{j}-\hat{k}$ and $\vec{b_2} = 2\hat{i}+\hat{j}+2\hat{k}$

$\vec{b_1}$ and $\vec{b_2}$ show direction to which l_1 and l_2 are parallel respectively.

since $\vec{b_1} \neq \vec{b_2}$, l_1 & l_2 may be skew lines,

and shortest d (d) between two skew lines is given by

$$d = \left| \frac{(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|} \right| \quad \text{—(1)}$$

now $\vec{a_2}-\vec{a_1} = (2-1)\hat{i}+(-1-2)\hat{j}+(-1-1)\hat{k}$

$= \hat{i}+(-3)\hat{j}+(-2)\hat{k}$

we calulate $\vec{b_1}\times\vec{b_2}$ using determinant

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$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2)$$

$$= -3\hat{i} + 0\hat{j} + 3\hat{k}$$

and $|\vec{b_1} \times \vec{b_2}| = \sqrt{3^2+0^2+3^2} = 3\sqrt{2}$

$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$

$$= (\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-3\hat{i} + 0\hat{j} + 3\hat{k})$$

$$= -3 + 0 - 6 = -9$$

put corresponding values in eqn (1)

$$d = \left| \frac{(-9)}{3\sqrt{2}} \right| = \frac{3}{\sqrt{2}} \text{ units.}$$

Hence shortest distance between l_1 & l_2 is $\frac{3}{\sqrt{2}}$ units.

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Question 21 'OR'

Given: $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} ; & x \neq \frac{\pi}{2} \\ 3 . & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

if any function $f(x)$ is continuous at c then

$$\boxed{\underset{\substack{x \to c^- \\ (LHL)}}{\lim} f(x) = f(c) = \underset{\substack{x \to c^+ \\ (RHL)}}{\lim} f(x).} \quad \text{---} (1)$$

now $LHL = \lim_{x \to c^-} f(x) = \lim_{h \to 0} f(c-h)$ (h is small positive no.)

here $\boxed{c = \frac{\pi}{2}}$

$= \lim_{h \to 0} \frac{k\cos(\frac{\pi}{2} - h)}{\pi - 2(\frac{\pi}{2} - h)}$

$= \lim_{h \to 0} \frac{k \sin h}{2h}$ $(\cos(90 - \theta) = \sin\theta)$.

$= \lim_{h \to 0} \frac{k}{2}\left(\frac{\sin h}{h}\right)$

$LHL = \frac{k}{2}$ $\left[\text{as } \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$

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in the same way

$RHL = \lim_{x \to c^+} f(x) = \cancel{\lim f(x)} \lim_{h \to 0} f(c+h)$

$= \lim_{h \to 0} \dfrac{k\cos(\frac{\pi}{2}+h)}{\pi - 2(\frac{\pi}{2}+h)}$ $\qquad (\cos(90+\theta) = -\sin\theta)$

$= \lim_{h \to 0} \dfrac{(k)-\sin h}{-2h}$

$= \lim_{h \to 0} \dfrac{k}{2}\left(\dfrac{\sin h}{h}\right)$ $\qquad \left[\lim_{x \to 0} \dfrac{\sin x}{x} = 1\right]$

$RHL = \dfrac{k}{2}$

from condition of continuity (eqn ①) we see,

$\dfrac{k}{2} = f(c) = \dfrac{k}{2}$ $\qquad (\therefore LHL = f(c) = RHL)$

$\dfrac{k}{2} = f(\frac{\pi}{2}) = \dfrac{k}{2}$

$\dfrac{k}{2} = 3$

$\boxed{k=6}$ Ans.

Hence required value of $\boxed{k=6}$

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Question 22

Let $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ — ①

now using the property :

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}$ $\left(\because \begin{array}{l} \sin(\pi - x) = \sin x \\ \cos^2(\pi - x) = \cos^2 x \end{array} \right)$

$I = \int_0^{\pi} \frac{\pi \sin x \, dx}{1+\cos^2 x} - \int_0^{\pi} \frac{x \sin x \, dx}{1+\cos^2 x}$ — ②

add eqn ① & ②

$2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} dx$

So $I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$.

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now, we will integrate using subsitution

put $\cos x = t$

then $-\sin x\,dx = dt$ (on differentiation)

or $\sin x\,dx = -dt$

now, changing limit according to subsitution

at $x=0$ and $x=\pi$

$t=\cos(0)=1$ $\quad t=\cos\pi=-1$ so

x	0	π
t	1	-1

so

$$I = \frac{\pi}{2}\int_{1}^{-1}\frac{-dt}{1+t^2} = \int_{-1}^{1}\frac{dt}{1+t^2}$$

$\left(\text{By propert } \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx\right)$

$$I = \frac{\pi}{2}\left[\tan^{-1}t\right]_{-1}^{1}$$

as $\left(\int\frac{1}{1+x^2}dx = \tan^{-1}x + C\right)$

$$I = \frac{\pi}{2}\left(\tan^{-1}(1) - \tan^{-1}(-1)\right)$$

$$= \left(\frac{\pi}{2}\right)\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$$

$\left[\begin{array}{l}\tan^{-1}(1) = \frac{\pi}{4}\\ \tan^{-1}(-1) = -\frac{\pi}{4}\end{array}\right]$

$$I = \left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right)$$

so $\boxed{I = \frac{\pi^2}{4}}$

It is the required value of integration

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Question 23

Given : $x\frac{dy}{dx}+2y=x^2$ — ① $(x \neq 0)$

To find : general solution for differential equation.

on rearranging eqn ①, we get

$$\boxed{\frac{dy}{dx}+\frac{2}{x}y=x}$$

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It is similar to general form of linear differential equation of 1^{st} order

$$\boxed{\frac{dy}{dx}+P(x)y=Q(x)}$$ general form

P(x) is function of x only (or constant)
Q(x) is also function of x only.

On comparing, we get

$P(x)=\frac{2}{x}$

and $Q(x)=x$.

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Integrating factor (I.F) = $e^{\int P(x)dx}$

so, I.F. = $e^{\int \frac{2}{x} dx}$

$= e^{2\ln x} = e^{\ln x^2}$

I.F. $\Rightarrow x^2$ (as i.e $e^{\ln a} = a$)

the solution is given by

$\Rightarrow y(I.F.) = \int (Q(x)(I.F.))dx + c$ (c is arbitary constant).

so,

$\Rightarrow y(x^2) = \int x(x^2)dx + c$

$\Rightarrow y(x^2) = \int x^3 dx + c$

$\Rightarrow y(x^2) = \frac{x^4}{4} + c$ (as $\int x^n dx = \frac{x^{n+1}}{n+1}$)

$y = \frac{x^2}{4} + \frac{c}{x^2}$

or $\boxed{y = \frac{x^2}{4} + cx^{-2}}$

Hence it the required general solution of given differential eqn.

THE END.

योग पूर्व पृष्ठ + पृष्ठ 31 के अंक = कुल अंक

माध्यमिक शिक्षा मण्डल मध्यप्रदेश

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See question no. 19 on page 21
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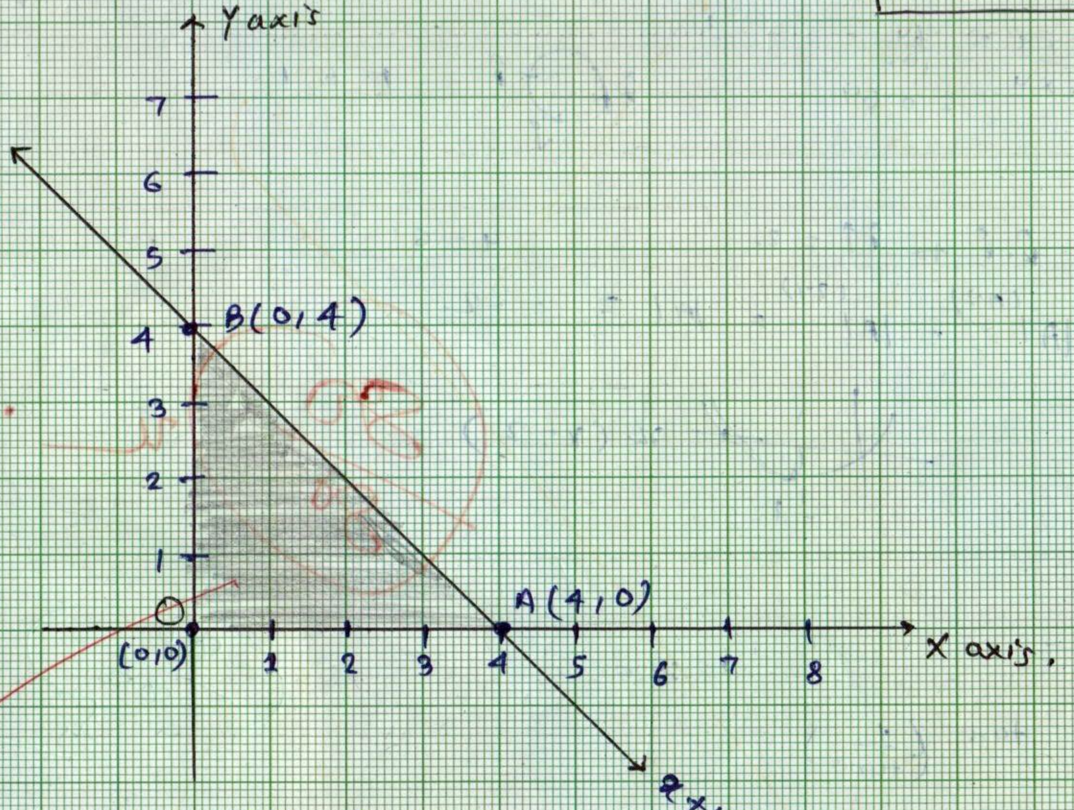
Question: 19 Graph → page 21

$\boxed{1cm = 1unit}$ on both X and Y axis.



region OAB is feasible region (and bounded).