



माध्यमिक शिक्षा मण्डल, मध्य प्रदेश, भोपाल  
परीक्षार्थी द्वारा भरा जावे ↓

2017

24 पृष्ठीय

परीक्षार्थी द्वारा भरा जावे ↓

स्टीकर तीर के निशान ↓ से मिलाकर लगाय

उत्तर पुस्तिका का सरल क्रमांक **A - 0897455**

अंकों में परीक्षार्थी का रोल नम्बर ✓

2	7	2	1	3	4	1	4	9	-
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शब्दों में ✓

नीचे दिये गये उदाहरण अनुसार रोल नम्बर भरें।

उदाहरणार्थ 

1	1	2	4	3	9	5	6	8
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एक एक दो चार तीन नौ पांच छः आठ

केन्द्राध्यक्ष / सहायक केन्द्राध्यक्ष एवं परीक्षक द्वारा भरा जावे ↓

क :- पूरक उत्तर पुस्तिकाओं की संख्या अंकों में **2** शब्दों में **two**

ख :- परीक्षार्थी का कक्ष क्रमांक **10**

ग :- परीक्षा का दिनांक **18 03 2017**

परीक्षा का नाम एवं परीक्षा केन्द्र क्रमांक की मुद्रा

**पर सेकेण्डरी सर्टी० परीक्षा केन्द्र क्रमांक-212027**

पर्यवेक्षक का हस्ताक्षर एवं केन्द्राध्यक्ष / सहायक / केन्द्राध्यक्ष के हस्ताक्षर

*(Signature)* *(Signature)*

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे ↓

परीक्षक एवं उपमुख्य परीक्षक द्वारा भरा जावे ↓

प्रमाणित किया जाता है कि मूल्यांकन के समय पूरक उत्तर पुस्तिकाओं की संख्या उपरोक्तानुसार सही पाई होलो क्राफ्ट स्टीकर क्षतिग्रस्त नहीं पाया गया तथा अन्दर के पृष्ठों के अनुरूप मुख्य पृष्ठ पर अंकों की प्रविष्टि एवं अंकों का योग सही है।

निर्धारित मुद्रा : नाम, पदनाम, मोबाईल नम्बर, परीक्षक क्रमांक एवं पदांकित संस्था के नाम की मुद्रा लगाएं।

उप मुख्य परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा परीक्षक के हस्ताक्षर एवं निर्धारित मुद्रा

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**D. AKHTAR**  
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केवल परीक्षक द्वारा भरा जावे।

प्रश्न क्रमांक के सम्मुख प्राप्तियों की प्रविष्टि करें।

प्रश्न क्रमांक	पृष्ठ क्रमांक	प्राप्तांक (अंको में)
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28		

कुल प्राप्तांक अंकों में

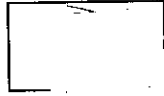
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2



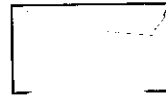
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पृष्ठ 2 क अंक

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प्रश्न क.

Q. 1)

0-1

(i)

(b) 2 ✓

(ii)

(d)  $\sin 1x$  ✓

(iii)

(a)  $26/3$  ✓

(iv) B

(a)  $\frac{\vec{a}}{|\vec{a}|}$  ✓

(v) S

(d)  $\cot x$  ✓

E

Q. 2.

Q-2

(i)

False ✓

(ii)

True ✓

(iii)

True ✓

(iv)

False ✓

(v)

True ✓

Rough

~~(a) cos 0 = 0~~  
cos 90°

$2x+3 \Rightarrow A(x+3) + B(x+2)$   
 $-4 \Rightarrow A(0) + (-1) + 3 = B(+)$   
 $B = -3$   
 $A(x+3) + B(x+2)$   
 $A+B = 2$

$\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}}$

$\tan^{-1} \left( \frac{\sin \theta}{\cos \theta} \right)$   
 $\frac{1}{1.69} = \frac{1}{1.3^2}$   
 $\sin^{-1} \frac{\sin \theta}{\cos \theta} = \tan^{-1} \frac{\sin \theta}{\cos \theta}$

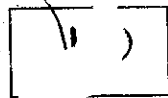
$\frac{1}{3} (27 - 1)$   
 $\frac{26}{3}$

$2x+y-1 = \sqrt{0.8 \times 0.2}$   
 $\frac{2x+y-1}{5} = \sqrt{\frac{0.16}{100}}$   
 $\frac{2x+y-1}{5} = \frac{4}{10} (0.4)$   
 $\frac{3}{2} = \frac{5}{2} (1)$   
 $\frac{4}{10} = 0.4$

$f'(x) = 5$   
cos x  
sin x

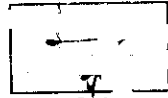
$\frac{0.8 \times 0.2}{10} = \frac{0.16}{10}$   
 $\frac{4}{10} = 0.4$   
 $\frac{2}{5} = 0.4$

3

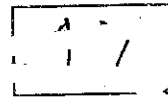


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Q-3

(i)  $\sqrt{194}$  units

(ii)  $Ax + By + Cz = 0$

(iii)  $5/2$  unit

(iv)  $(-3/2, -5/2, -1)$

$\cos 3x$

5, 12, 13  
↓  
(0, 12, 0)

$$\sqrt{25 + 169}$$

$$169$$

$$25$$

$$\sqrt{194}$$

$$2x + y - z = 5$$

$$\frac{2x}{5} + \frac{y}{5} + \frac{z}{-5} = 1$$

$x/5/2$

$-3/2, -5/2, -1$

$$\frac{d}{dx} \sin 3x$$

$$\cos 3x \cdot \frac{3}{3}$$

Q-4

(i) Formula for finding square root of the Number N

$$x_{n+1} = \frac{1}{2} \left[ x_n + \frac{N}{x_n} \right]$$

(ii) Trapezoidal rule

$$\int_a^b f(x) dx = \frac{h}{2} [y_0 + y_n + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})]$$

here  $h = \frac{b-a}{n}$

**B  
S  
E**

Q.4

4

$$\boxed{\phantom{00}} + \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

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(iii)

$$\sqrt[3]{10} = \underline{\underline{2.167}}$$

$$2 \leq \sqrt[3]{10} \leq 3$$

$$x_1 = \frac{1}{3} \left[ 2 + \frac{4}{2} + \frac{10}{4} \right]$$

$$\frac{1}{3} \left[ \frac{16+10}{4} \right] = \frac{26}{12} = \frac{13}{6}$$

$$x_1 = \frac{1}{3} \left[ 2x_0 + \frac{N}{(x_0)^2} \right]$$

$$\begin{array}{r} 2.167 \\ 6 \overline{) 13} \\ \underline{12} \\ 10 \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

2.167

$$x_1 = \frac{1}{3} \left( 2 + \frac{10}{2} \right) = \frac{13}{6}$$

$2x_3 + x_1 - 3 = 0$   
 $p_0 \rightarrow -3$   
 $f(1) = 1 + 1 - 3 = -1$   
 $f(2) = 8 + 2 - 3 = 7$

B  
S  
E

(iv)

$$\text{Interval} = (1, 2)$$

$$f(x) = x^3 + x - 3$$

$$f(0) = -3$$

$$f(1) = 1 + 1 - 3 = -1$$

$$f(2) = 8 + 2 - 3 = 7$$

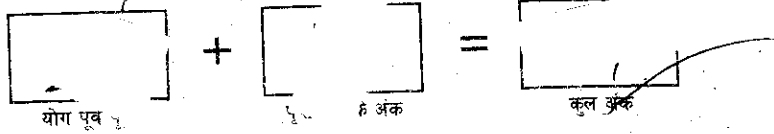
(7)

(v)

The coefficient of  $y$  with odd subscripts in Simpson's rule

$$= \boxed{4}$$

5



A

B

- a)  $\int \sec x \, dx$  (iv)  $\log \tan \frac{x}{2} + c$
- b)  $\int \frac{dx}{x\sqrt{x^2-1}}$  (i)  $\sec^{-1} x + c$
- c)  $\int \sqrt{a^2-x^2} \, dx$  (ii)  $\frac{1}{2} \left[ x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right] + c$
- d)  $\int \frac{dx}{\sqrt{a^2-x^2}}$  (v)  $\sin^{-1} \frac{x}{a} + c$
- e)  $\int \frac{dx}{a^2+x^2}$  (vi)  $\frac{1}{a} \tan^{-1} \frac{x}{a} + c$

Section - B

Question - No - 6

or

Solve  $\rightarrow$  (6) Given position vector of points A and B are -

$$\vec{OA} = 7\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{OB} = 2\hat{i} + 5\hat{j} + 4\hat{k}$$

$$\vec{AB} = \text{P.V of B} - \text{P.V of A}$$

$$= \vec{OB} - \vec{OA}$$

$$= 2\hat{i} + 5\hat{j} + 4\hat{k} - 7\hat{i} - 3\hat{j} - \hat{k}$$

$$= -5\hat{i} + 2\hat{j} + 3\hat{k}$$

$$|\vec{AB}| = \sqrt{(-5)^2 + (2)^2 + (3)^2} = \sqrt{25+4+9}$$

$$= \sqrt{38} = \underline{\underline{\sqrt{38}}} \text{ ans}$$

25  
4  
9  
38

B  
S  
E

6

$$\left[ \begin{array}{c} \phantom{a} \\ \phantom{b} \\ \phantom{c} \end{array} \right] + \left[ \begin{array}{c} \phantom{a} \\ \phantom{b} \\ \phantom{c} \end{array} \right] = \left[ \begin{array}{c} \phantom{a} \\ \phantom{b} \\ \phantom{c} \end{array} \right]$$



प्रश्न क.

Question - No-7.

Solve (7) Given force  $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$

its displacement  $\vec{d} = 3\hat{i} + 2\hat{j} - 5\hat{k}$

To find work done

We know work =  $W = \vec{F} \cdot \vec{d}$

B  
S  
E

$$\text{Work} = (2\hat{i} - \hat{j} - \hat{k}) \cdot (3\hat{i} + 2\hat{j} - 5\hat{k})$$

$$\begin{aligned} \text{Work} &= 6 - 2 + 5 \\ &= \underline{\underline{9 \text{ units}}} \end{aligned}$$

Question - No-8.

Or

Solve: (8):

The shortest distance between two straight lines whose vector equations are

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \quad \text{--- (1)}$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$$



8

$$\boxed{\phantom{x}} + \boxed{\phantom{x}} = \boxed{\phantom{x}}$$



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Question NO - 10

Solve :- (10) Value of  $\int x e^x dx$

$$\int x e^x dx =$$

We know that

$$\int u \cdot v dx = u \int v dx - \int \left[ \frac{du}{dx} \int v dx \right] dx$$

$$\text{Here } u = x, \\ v = e^x$$

$$\int x e^x dx = x \int e^x dx - \int \frac{dx}{dx} \int e^x dx$$

$$= x e^x - \int 1 \cdot e^x dx$$

$$= \underline{\underline{x e^x - e^x}} \quad [\because \int e^x dx = e^x]$$

Question - 11

Solve :- (11) Given parallel planes are

$$2x - 2y + z + 3 = 0$$

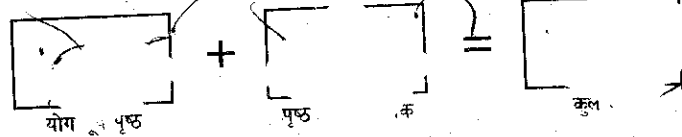
~~4x~~

Multiplying by 2 both side, we get

$$4x - 4y + 2z + 6 = 0 \quad \text{--- (1)}$$



9



and  $4x - 4y + 2z + 5 = 0$  — (2)

We know if planes are parallel, distance between them :

$$\text{Distance} = \left| \frac{d_2 - d_1}{\sqrt{A^2 + B^2 + C^2}} \right|$$

Here  $d_1 = 5$  ,  $d_2 = 6$

$A = 4$  ,  $B = 4$  ,  $C = 2$

$$\therefore \text{Distance} = \left| \frac{6 - 5}{\sqrt{(4)^2 + (4)^2 + (2)^2}} \right|$$

$$= \frac{1}{\sqrt{16 + 16 + 4}}$$

$$= \frac{1}{\sqrt{36}}$$

$$\text{Distance} = \boxed{\frac{1}{6} \text{ units}}$$

Rough

$$2x - 2y + z + 3 = 0$$

$$2x - 2y + z + 5 = 0$$

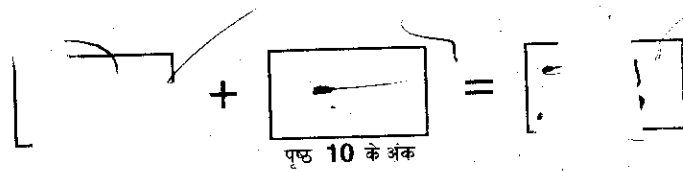
$$3 - 5/2$$

$$\sqrt{4 + 4 + 1}$$

$$\frac{1/2}{3}$$

$$\frac{1}{6}$$

10



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Question - 12.

Solve (12)

To prove :-

The symmetrical form of equations

$$x = ay + b$$

$$z = cy + d$$

~~(12)~~

$$\text{is } \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

**B  
S  
E**

Given equations are

$$x = ay + b \quad \text{--- (1)}$$

$$\text{and } z = cy + d \quad \text{--- (2)}$$

from (1)

$$x = ay + b$$

$$x - b = ay$$

$$\frac{x-b}{a} = \frac{y}{1} \quad \text{--- (3)}$$

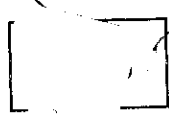
from (2)

$$z = cy + d$$

$$z - d = cy$$

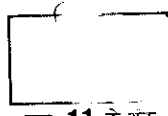
$$\frac{z-d}{c} = \frac{y}{1} \quad \text{--- (4)}$$

(11)

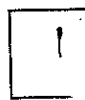


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from (3) and (4)

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

which is the symmetrical form of equation.  
Hence proved!

Question No - (13)

(13)

To prove

$$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$$

Taking LHS

$$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$$

$$\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$$

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{a} + \vec{a} \times \vec{c} + \vec{c} \times \vec{a} + \vec{b} \times \vec{c} + \vec{c} \times \vec{b}$$

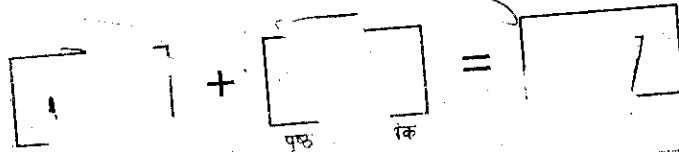
$$\because \begin{cases} \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a}) \\ \vec{a} \times \vec{c} = -(\vec{c} \times \vec{a}) \\ \vec{b} \times \vec{c} = -(\vec{c} \times \vec{b}) \end{cases}$$

$$\Rightarrow -(\vec{b} \times \vec{a}) + (\vec{b} \times \vec{a}) - (\vec{c} \times \vec{a}) + \vec{c} \times \vec{a} - (\vec{c} \times \vec{b}) + (\vec{c} \times \vec{b})$$

$$\Rightarrow \vec{0} \text{ (RHS)}$$

Hence proved

12



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Question No → 14 :

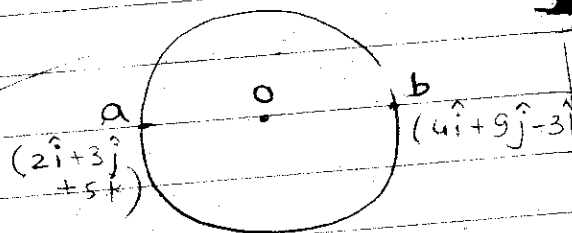
Solve (14)

Vector equation of the sphere when points  $(2, 3, 5)$  and  $(4, 9, -3)$  are extremities of its diameter.

Given

P.V of  $\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

P.V of  $\vec{b} = 4\hat{i} + 9\hat{j} - 3\hat{k}$



**B  
S  
E**

We know vector equation of sphere whose end points are given is given by

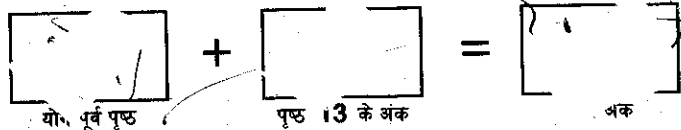
$$(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = 0$$

$$[\vec{r} - (2\hat{i} + 3\hat{j} + 5\hat{k})] \cdot [\vec{r} - (4\hat{i} + 9\hat{j} - 3\hat{k})] = 0$$

$$(\vec{r} - 2\hat{i} - 3\hat{j} - 5\hat{k}) \cdot (\vec{r} - 4\hat{i} - 9\hat{j} + 3\hat{k}) = 0$$

which is the required vector equation

Question 1



### Question - NO - 15.

(15) Resolve  $\frac{13x + 18}{2x^2 + 5x + 3}$  into partial fractions

~~Let~~ given  $\frac{13x + 18}{2x^2 + 5x + 3} = \frac{13x + 18}{(x+1)(2x+3)}$

Let  $\frac{13x + 18}{(x+1)(2x+3)} = \frac{A}{x+1} + \frac{B}{2x+3}$  - (1)

$$\frac{13x + 18}{(x+1)(2x+3)} = \frac{A(2x+3) + B(x+1)}{(x+1)(2x+3)}$$

$$13x + 18 = A(2x+3) + B(x+1)$$

Now  ~~$x = -1, x = -3$~~

Now  $x + 1 = 0 \Rightarrow x = -1$

and  $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

When  $x = -1$

$$13(-1) + 18 = A(2(-1) + 3)$$

$$-13 + 18 = A(-2 + 3)$$

$$5 = A(1)$$

$$\boxed{A = 5}$$

14

$$\left[ \frac{1}{x+1} \right] + \left[ \frac{1}{2x+3} \right] = \left[ \frac{\text{कुल अंक}}{\dots} \right]$$



प्रश्न क.

When  $x = \frac{-3}{2}$

$$13\left(\frac{-3}{2}\right) + 18 = B\left(\frac{-3}{2} + 1\right)$$

$$\frac{-39}{2} + 18 = B\left(\frac{-1}{2}\right)$$

$$\frac{-39 + 36}{2} = B\left(-\frac{1}{2}\right)$$

$$\frac{-3}{2} = B\left(-\frac{1}{2}\right)$$

$$B = 3$$

B  
S  
E

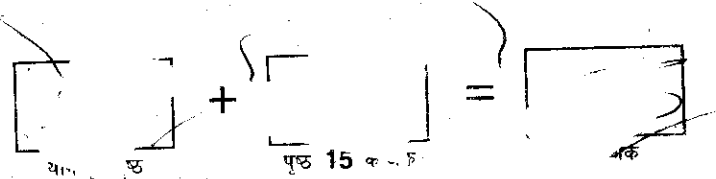
Putting the values of A and B in eq-①

$$\frac{13x + 18}{(x+1)(2x+3)} = \frac{5}{x+1} + \frac{3}{2x+3}$$

$$\frac{13x + 18}{2x^2 + 5x + 3} = \frac{5}{x+1} + \frac{3}{2x+3}$$

~~From~~  
 $\int x^2 dx = \frac{1}{3}x^3 + C$   
 $\int \frac{1}{x} dx = \ln|x| + C$   
 $\int \frac{1}{2x-1} dx = \frac{1}{2} \ln|2x-1| + C$   
 $\frac{13x+18}{10x^2+15x+3} = \frac{A}{x+1} + \frac{B}{2x+3}$

15



Question NO - 16 :

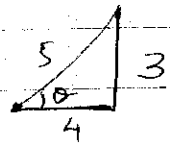
we → (16) To prove that

$$\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{5} = \tan^{-1} \frac{27}{11}$$

Taking LHS

$$\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{5}$$

$$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5}$$



$$= \tan^{-1} \left[ \frac{\frac{3}{4} + \frac{3}{5}}{1 - \frac{3}{4} \times \frac{3}{5}} \right]$$

$$\therefore \cos^{-1} \frac{4}{5} = \tan^{-1} \frac{3}{4}$$

$$= \tan^{-1} \left[ \frac{15 + 12}{20} \right] / \left[ \frac{20 - 9}{20} \right]$$

$$\left[ \begin{aligned} &\tan^{-1}(x) + \tan^{-1}(y) \\ &= \tan^{-1} \left( \frac{x+y}{1-xy} \right) \end{aligned} \right]$$

Here  $x = 3/4$

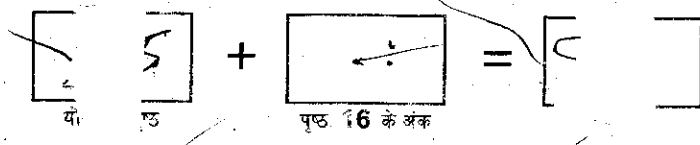
$y = 3/5$

$$= \tan^{-1} \left[ \frac{27/20}{11/20} \right]$$

$$= \tan^{-1} \left( \frac{27}{11} \right) \text{ (RHS)}$$

Hence LHS = RHS

Hence proved



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Question NO - 17

same 17

~~To solve~~

$$\frac{dy}{dx} =$$

To find ~~the~~ differential coefficient of  $\sqrt{\tan x}$ .

Given  $y = \sqrt{\tan x}$

differentiate with respect to  $x$ .

$$\frac{dy}{dx} = \frac{d}{dx} \sqrt{\tan x}$$

Let  $\tan x = t$

$$\frac{dy}{dx} = \frac{d}{dx} \sqrt{t}$$

$$= \frac{1}{2\sqrt{t}} \frac{d}{dx} (t)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan x}} \frac{d}{dx} \tan x$$

Let  $\tan x = m$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\tan x}} \frac{d}{dx} \tan m$$

$$= \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 m \cdot \frac{dm}{dx}$$

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$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan \sqrt{x}}} \cdot \sec^2 \sqrt{x} \cdot \frac{d\sqrt{x}}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan \sqrt{x}}} \cdot \sec^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$\left[ \because \frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}} \right]$$

$$\left[ \because \frac{d \tan x}{dx} = \sec^2 x \right]$$

### Question - 18

Solve - 18

To prove that

$$(1 - 2y) \frac{dy}{dx} = \sin x$$

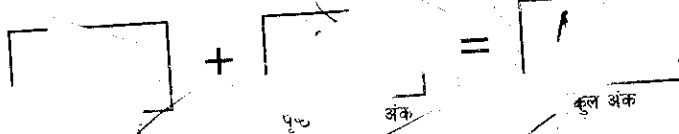
Given  $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$

Then

$$y = \sqrt{\cos x + y}$$

Squaring both side

18



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$$y^2 = \cos x + y$$

Differentiate with respect to  $x$ .

$$\frac{d}{dx} y^2 = \frac{d}{dx} \cos x + \frac{dy}{dx}$$

$$2y \frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\sin x = \frac{dy}{dx} (1 - 2y)$$

$$\therefore \frac{dy}{dx} =$$

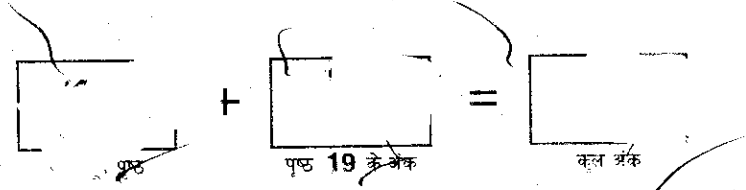
$$\therefore \boxed{(1 - 2y) \frac{dy}{dx} = \sin x}$$

Hence proved

$$\therefore \left[ \frac{d}{dx} y^2 = 2y \frac{dy}{dx} \right]$$

$$\therefore \left[ \frac{d}{dx} \cos x = -\sin x \right]$$

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Question - 19

Q. 19

Given side of a square sheet of metal is increasing at the rate of 5 cm/minute.

Let the side of square sheet be  $a$

$$\therefore \frac{da}{dt} = 5 \text{ cm/minute}$$

To find  $\frac{dA}{dt}$  when the side <sup>(a)</sup> is 20 cm.

We know Area of square sheet

$$A = \text{side} \times \text{side}$$

$$A = a^2$$

$$\therefore \frac{dA}{dt} = \frac{d}{dt} a^2$$

$$= 2a \frac{da}{dt}$$

$$= 2 \times 20 \times 5$$

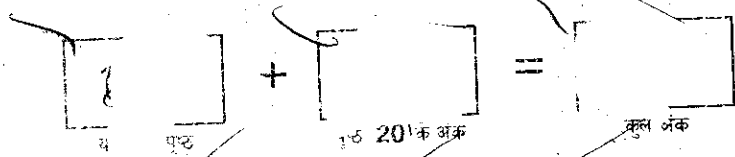
[ $\because a = 20 \text{ cm}$   
 $\therefore \frac{da}{dt} = 5 \text{ cm/minute}$ ]

$$= \boxed{200 \text{ cm}^2/\text{minute}}$$

Hence when side is 20 cm long its area is increasing at the rate of 200 cm<sup>2</sup>/minute

$$\frac{dA}{dt} = 2 \times 20 \times 5 = 200 \text{ cm}^2/\text{min}$$

20



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Question NO - 20

Solve: - (20) To find  $\text{cov}(X, Y)$

Given

$$\begin{aligned} \sum x_i &= 15 \\ \sum y_i &= 36 \\ \sum x_i y_i &= 110 \\ n &= 5 \end{aligned}$$

We know that

$$\text{Covariance}(X, Y) = \frac{\sum xy}{n} - \left( \frac{\sum x}{n} \cdot \frac{\sum y}{n} \right)$$

$$\text{Cov}(X, Y) = \frac{110}{5} - \left( \frac{15}{5} \times \frac{36}{5} \right)$$

$$= 22 - 3 \times (7.2)$$

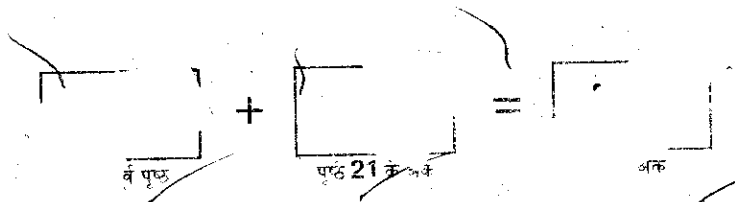
$$= 22 - 21.6$$

$$\text{Cov}(X, Y) = \underline{\underline{0.4}}$$

$$\begin{array}{r} 7.2 \\ \times 3 \\ \hline 21.6 \end{array}$$

$$\begin{array}{r} 110 \\ \times 5 \\ \hline 550 \end{array}$$

21



Question No - 21

To prove correlation coefficient ( $r$ ) is the geometric mean of the regression coefficients.

To prove  $r = \sqrt{b_{yx} \cdot b_{xy}}$

We know that

$b_{yx}$  = regression coefficient of  $y$  on  $x$

$$b_{yx} = r \frac{\sigma_y}{\sigma_x} \quad \text{--- (1)}$$

and regression coefficient of  $x$  on  $y$  is

$$b_{xy} = r \frac{\sigma_x}{\sigma_y} \quad \text{--- (2)}$$

Multiplying (1) and (2)

(1) x (2)

$$\Rightarrow b_{xy} \cdot b_{yx} = r^2 \cdot \frac{\sigma_y}{\sigma_x} \times \frac{\sigma_x}{\sigma_y}$$

$$\Rightarrow r^2 = b_{xy} \cdot b_{yx}$$

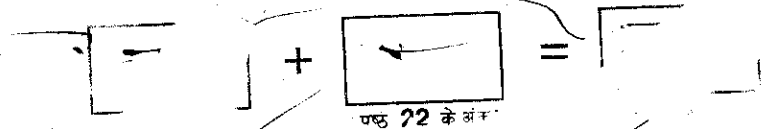
Taking square root

$$r = \sqrt{b_{yx} \cdot b_{xy}}$$

Hence proved

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### Question No - 22

Solve (22) To prove that the lines  
 $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$  and

$$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} \text{ are coplanar}$$

and also find point of intersection of these lines.

Given,  
 Equations of lines are

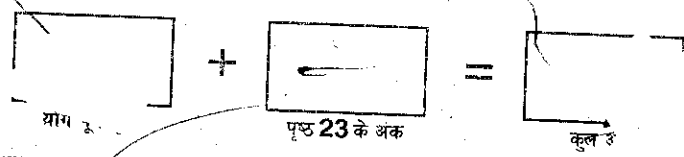
$$\frac{x-0}{1} = \frac{y-2}{2} = \frac{z-(-3)}{3} \quad \text{--- (1)}$$

$$\text{and } \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} \quad \text{--- (2)}$$

Comparing these lines with general eq. of lines, we get

$x_1 = 0$	$y_1 = 2$	$z_1 = -3$
$x_2 = 2$	$y_2 = 6$	$z_2 = 3$
$a_1 = 1$	$b_1 = 2$	$c_1 = 3$
$a_2 = 2$	$b_2 = 3$	$c_2 = 4$

23



We know if lines are coplanar

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Taking LHS

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

Expanding with respect to R<sub>1</sub>

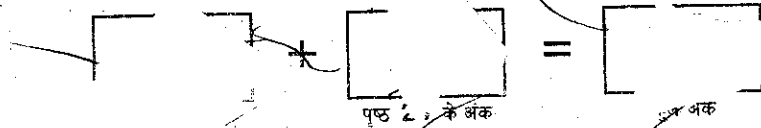
$$\begin{aligned} &= 2(8 - 9) - 4(4 - 6) + 6(3 - 4) \\ &= 2(-1) - 4(-2) + 6(-1) \\ &= -2 + 8 - 6 \\ &= 0 \text{ (RHS)} \end{aligned}$$

Hence LHS = RHS

∴ Given lines are coplanar.

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24



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For intersecting point  
from (1) line

$$\frac{x-0}{1} = \frac{y-2}{2} = \frac{z+3}{3} = k$$

$$x = k, \quad y = 2k + 2, \quad z = 3k - 3 \quad \text{--- (3)}$$

which are coordinates of intersecting point

Putting the value of  $x$  and  $y$  in eq (2)

$$\frac{k-2}{2} = \frac{2k+2-6}{3}$$

$$\frac{k-2}{2} = \frac{2k-4}{3}$$

$$3k - 6 = 4k - 8$$

$$3k - 4k = 6 - 8$$

$$+k = +2$$

$$\boxed{k = 2}$$

Putting the value of  $\boxed{k = 2}$  in eq (3)

we get

$$x = k = 2$$

$$y = 2k + 2 = 2 \times 2 + 2 = 6$$

$$z = 3k - 3 = 3 \times 2 - 3 = 3$$

$\therefore (x, y, z)$  or its intersecting points is  $\underline{\underline{(2, 6, 3)}}$

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4 पृष्ठीय

परीक्षार्थी द्वारा भरा जावे ↓

परीक्षा का विषय

विषय कोड

परीक्षा का माध्यम

परीक्षा का दिनांक

18 03 2017

परीक्षा का नाम एवं परीक्षा केन्द्र क्रमांक की नुदा  
**हायर सेकेण्डरी सर्टीफिकेट परीक्षा**  
**केन्द्र क्रमांक-212027**

पर्यवेक्षक का नाम *म. प्र. बो.*  
*(म. प्र. बो.)*

केन्द्राध्यक्ष/सहायक केन्द्राध्यक्ष के हस्ताक्षर

मुख्य उत्तर पुस्तिका के अंतिम पृष्ठ क्रमांक.....तक कुल प्राप्तांक  +  =

Question No - 23

To prove  $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$

Given

$$f(x) = \log_e \left( \frac{1-x}{1+x} \right)$$

Taking LHS

$$f(a) + f(b)$$

Now

$$f(a) = \log_e \left( \frac{1-a}{1+a} \right) \quad - (1)$$

②  $\square + \square = \square$

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$$f(b) = \log_e \left( \frac{1-b}{1+b} \right) \quad \text{--- (2)}$$

Adding (1) + (2)

$$f(a) + f(b) = \log_e \left( \frac{1-a}{1+a} \right) + \log_e \left( \frac{1-b}{1+b} \right)$$

$$= \log_e \left( \frac{1-a}{1+a} \times \frac{1-b}{1+b} \right)$$

$$= \log_e \left( \frac{(1-a)(1-b)}{(1+a)(1+b)} \right)$$

$$f(a) + f(b) = \log_e \left( \frac{1-b-a+ab}{1+a+b+ab} \right) \quad \text{--- (3)}$$

Taking RHS

$$f\left(\frac{a+b}{1+ab}\right) = \log_e \left[ \frac{1 - \left(\frac{a+b}{1+ab}\right)}{1 + \left(\frac{a+b}{1+ab}\right)} \right]$$

$$= \log_e \left[ \frac{1+ab-a-b}{1+ab} \cdot \frac{1+ab}{1+ab+a+b} \right]$$

$$f\left(\frac{a+b}{1+ab}\right) = \log_e \left[ \frac{1-a-b+ab}{1+a+b+ab} \right] \quad \text{--- (4)}$$

3

$$\boxed{\phantom{000}} + \boxed{\phantom{000}} = \boxed{\phantom{000}}$$

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from (3) and (4)

$$\boxed{f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)}$$

Hence proved.

Question No - 24.

To prove that

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x} \, dx}{1 + \sqrt{\tan x}} = \frac{\pi}{4}$$

Taking L.H.S

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x} \, dx}{1 + \sqrt{\tan x}}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}{1 + \frac{\sqrt{\sin x}}{\sqrt{\cos x}}} \, dx$$

4

$$\boxed{\sqrt{\sin x}} + \boxed{\sqrt{\cos x}} = \boxed{\sqrt{\sin x + \cos x}}$$

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$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{--- (1)}$$

Now

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}} dx$$

Here  $a = \pi/2$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \text{--- (2)}$$

Adding (1) and (2)

$$(1+2) \Rightarrow$$

$$2I = \int_0^{\pi/2} \left( \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$$

$$2I = \int_0^{\pi/2} 1 dx \quad \int 1 dx = x$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$



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4 पृष्ठीय

परीक्षार्थी द्वारा भरा जावे ↓

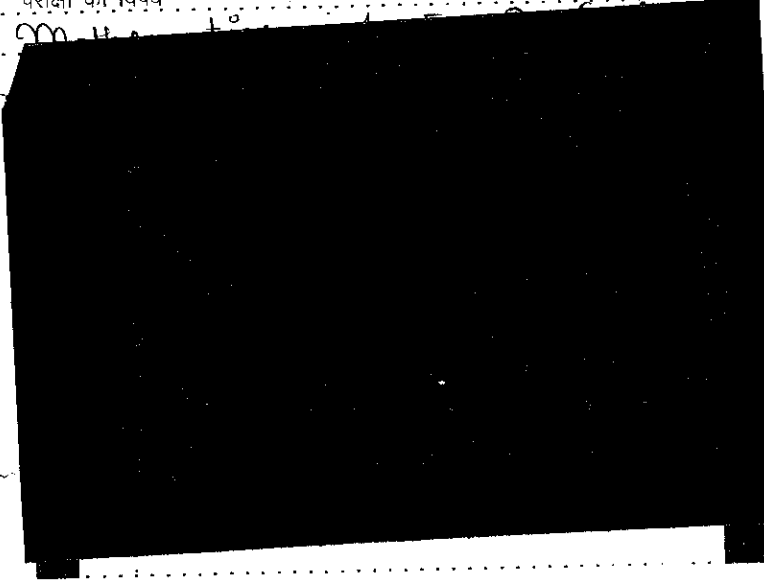
परीक्षा का विषय

विषय कोड

परीक्षा का माध्यम

परीक्षा का दिनांक

18 03 2017



परीक्षा का नाम एवं परीक्षा केन्द्र क्रमांक की मुद्रा

**हायर सेकेण्डरी सर्टी० परीक्षा**

केन्द्र क्रमांक - 212027

पर्यवेक्षक का नाम एवं हस्ताक्षर

(*रविशंकर*)

केन्द्राध्यक्ष/सहायक केन्द्राध्यक्ष के हस्ताक्षर

(*[Signature]*)

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$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx = \frac{\pi}{4}$$

Hence proved.

Question No - 25.

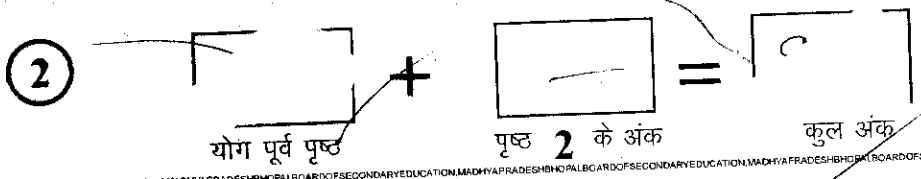
(25) Given  $\frac{dy}{dx} = \frac{x^2 + 5xy + 4y^2}{x^2}$  (1)

which is a homogeneous equation in degree - 2.

Let  $y = vx \Rightarrow v = \frac{y}{x}$

Differentiate with respect to x.

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad (2)$$



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from (1)

$$\frac{dy}{dx} = \frac{x^2 + 5x(vx) + 4v^2x^2}{x^2}$$

$$v + x \frac{dv}{dx} = \frac{x^2 + 5vx^2 + 4v^2x^2}{x^2}$$

$$v + x \frac{dv}{dx} = 1 + 5v + 4v^2$$

$$x \frac{dv}{dx} = 1 + 5v + 4v^2 - v$$

$$x \frac{dv}{dx} = 4v^2 + 4v + 1$$

$$x \frac{dv}{dx} = (2v + 1)^2$$

$$\frac{dv}{(2v + 1)^2} = \frac{dx}{x}$$

$\int \frac{dv}{(2v+1)^2}$   
 $\int \frac{dv}{2(2v+1)}$   
 $\frac{1}{2} \int \frac{dv}{2v+1}$

Variables are separable,

∴ Integration both side

$$\int \frac{dv}{(2v+1)^2} = \int \frac{dx}{x} + c$$

$$\int \frac{dv}{(2v+1)^2} = \log_e x + c$$

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Now let  $2v + 1 = t$

Differentiating with respect to  $v$ .

$$2 \frac{dv}{dv} + \frac{d(t)}{dv} = \frac{dt}{dv}$$

~~2 + 0 =~~  $\frac{dt}{dv}$

$$2 dv = dt$$

$$dv = \frac{dt}{2}$$

$$\frac{1}{2} \int \frac{1}{t^2} dt = \log_e t + c$$

$$\frac{1}{2} \int t^{-2} dt = \log_e t + c$$

$$-\frac{1}{2} (t^{-1}) = \log_e t + c$$

$$-\frac{1}{2} \left( \frac{1}{2v+1} \right) = \log_e t + c$$

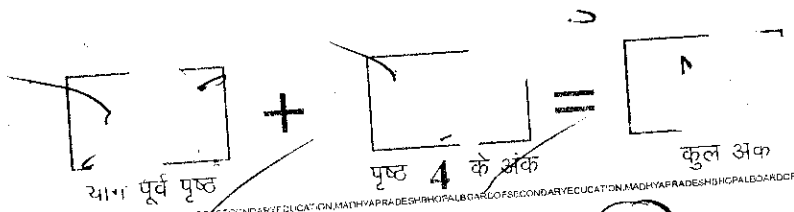
$$-\frac{1}{2} \left( \frac{1}{2(y/x)+1} \right) = \log_e t + c \quad [v = y/x]$$

$$-\frac{1}{2} \left( \frac{x}{2y+x} \right) = \log_e t + c$$

$$\frac{-x}{2(2y+x)} = \log_e t + c$$

which is the solution of the given differential equation.

4



Question - 26

Solve - (26) Given  
 a card is drawn at random from a well shuffled pack of 52 cards.

To find that it is neither <sup>an</sup> ace nor a king

Let  $P(A)$  denote it is neither an ace nor a king.

Then  $P(\bar{A})$  denote it is either an ace or a king.

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if it is either an ace or a king

$$n(\bar{A}) = 8$$

$$\text{and } n(S) = 52$$

$$P(\bar{A}) = \frac{n(\bar{A})}{n(S)}$$

$$P(\bar{A}) = \frac{8}{52}$$

$$\therefore P(A) = 1 - P(\bar{A})$$

$$= 1 - \frac{8}{52}$$

$$= \frac{52 - 8}{52} = \frac{44}{52} = \frac{11}{13}$$

$$\therefore P(A) = \frac{11}{13}$$

$\therefore$  Probability that it is neither an ace nor king  
 $P(A) = \left(\frac{11}{13}\right)$